Pension Reform for an Aging Japan: Welfare and Demographic Dynamics [*]

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Abstract

Life expectancy in Japan exceeds that in other developed countries; however, the standard starting age for receiving public pension benefits remains at 65 years. This paper uses an extended lifecycle general equilibrium model with endogenous fertility to investigate how a rise in the starting age for pension benefits will affect individual welfare and future demographic dynamics. Our simulation analysis indicates that per-capita welfare will increase if the starting age for pension benefits is raised to 68 or 70. The higher the employment rate of individuals aged 65 and older, the higher the per-capita welfare and the future population level. Conversely, when the employment rate for elderly individuals remains at 50% (Japan's current level), the total population would decrease in the long run, compared to the benchmark case with the normal pensionable age of 65. This paper demonstrates the importance of enhancing the employment rate of elderly individuals while raising the standard pensionable age in Japan.

Keywords: Public pension; welfare; demographic dynamics; Japanese economy; simulation analysis

JEL classification: H30; C68

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1. Introduction

Many developed countries currently face problems related to aging populations. Many countries, such as Germany, France, the United Kingdom, and the United States (U.S.), have already decided to raise the standard age at which people begin receiving public pension benefits to accommodate longer life spans; however, although Japan has a longer life expectancy than other advanced countries, the standard age for pension benefits is still 65 years. Balancing public pension finance would necessitate raising the starting age for pension benefits in Japan from 65 to, e.g., 68 or 70 years. When the average starting age rises, it may be crucial for individuals aged 65 and above to secure employment. According to the Ministry of Internal Affairs and Communications (2023a), the employment rate for people aged 65 to 69 in Japan was 50.8% in 2022, which has increased recently. An increasing number of seniors remain in Japan's workforce to compensate for the dwindling numbers of people in the working-age population. In Japan, a new law (the amended Act on Stabilization of Employment of Elderly Persons) came into effect in April 2021, which prescribes employment until 70 years of age as one's duty to make efforts. The measure aims to compensate for the growing labor shortage in an aging and shrinking population by encouraging elderly individuals to work longer and help support the social security system under demographic pressure.

Our model can evaluate public pension reforms from two viewpoints: individual welfare and future demography. We use the lifecycle general equilibrium simulation model of overlapping generations, developed by Auerbach and Kotlikoff (1983a, 1983b) and applied in Auerbach and Kotlikoff (1987), Auerbach et al. (1989), Altig et al. (2001), Homma et al. (1987), Ihori et al. (2006, 2011), and Okamoto (2013, 2020, 2021, 2022, 2024).[1] Using an extended Auerbach–Kotlikoff dynamic simulation model, we investigate the quantitative effects of a rise in the normal starting age for public pension benefits on per-capita welfare and future population.

The simulation model in Okamoto (2020, 2021) introduced the number of children freely chosen by households into the utility function, thus incorporating endogenous fertility and future demographic dynamics. Furthermore, Okamoto (2022, 2024) incorporated two representative households into a cohort: the low-income class (high school graduates) and the high-income class (university graduates). Okamoto

[1] The life-cycle model is considerably applicable to the Japanese economy. According to Horioka (2021), almost all of the available evidence suggests that the selfish life-cycle model applies, to some extent, in all countries and that there is more consistent support for this model in Japan than in the United States and other countries.

(2022, 2024) introduced the descendent link between a parent and children in the extended framework with endogenous fertility. This approach provided the exogenous transition probabilities from the parent's income class to the same (or the other) income class to which their children would belong. In other words, Okamoto's (2022, 2024) simulation model with endogenous fertility incorporated the descendent income inequality from parents to their children.

This paper's analytical model is based on Okamoto (2022, 2024); we extended the model to freely increase the standard starting age for receiving public pension benefits and to change the employment rate for elderly individuals. Our study can analyze these impacts on per-capita utility and future population dynamics. We quantitatively analyze how increasing the starting age for receiving public pension benefits in Japan impacts the future population levels and the welfare of all generations, including the future and the current generations. Specifically, we examine the effect of standard starting ages and employment rates for elderly individuals on the per-capita utility and the demographic dynamics for the transition process from 2022 to 2300. Thus, this paper analyzes the long-run impact on economic growth, welfare, and population levels, assuming alternative standard starting ages on pensions and employment rates for elderly individuals.

Finally, this study introduces an additional government institution, the Lump Sum Redistribution Authority (LSRA). Rises of the standard starting age for receiving public pension benefits generally improve the welfare of some generations but reduce that of others. If combined with redistribution from winning to losing generations, such changes may offer the prospect of *Pareto improvements*; however, without implementing intergenerational redistribution, potential efficiency gains or losses cannot be estimated. Therefore, like Auerbach and Kotlikoff (1987) and Nishiyama and Smetters (2005), we introduce the LSRA as a hypothetical government institution that distinguishes potential efficiency gains/losses from possible offsetting changes in the welfare of different generations. To isolate pure efficiency gains or losses, we consider simulation cases via LSRA transfers where the normal starting age for receiving public pension benefits is raised to 68 and 70. The introduction of LSRA transfers enables us to examine policy proposals from a long-term perspective, considering the welfare of current and future generations. Because of its ability to quantify alternative policies from a long-term perspective, we can present concrete and valuable policy proposals.

To investigate the abovementioned issue, we first examine the projected trend of Japan's aging population and then explain Japan's public pension system compared to other advanced countries.

Finally, we address the recent trend in the employment rate for elderly individuals in Japan.

1.1. Demographics in Japan

Figure 1 illustrates Japan's population data and projections from 1990 to 2100. Actual results on the total population until 2022 are based on the data from the Statistics Bureau of Japan (2023a). Projections after 2022 are based on data (by medium assumptions on fertility and mortality rates) from the National Institute of Population and Social Security Research (2023). The population increased monotonically until 2008, but the trend has reversed since 2008; the number of deaths exceeds births, and the total population will continue to decrease throughout the rest of the century. According to the official projection, by 2100, the population will reach 62.8 million, approximately half (50.2%) of the level in 2022 (124.9 million). Low fertility rates and a shrinking number of young females who can bear children contribute to the ongoing decline in the Japanese population.

Simultaneously, Japan's population is aging at an unprecedented speed for a developed nation. Figure 2 presents historical and projected aging rates (defined as the ratio of the population aged 65 or older against the total population) since 1990 for five major advanced countries. The data are based on the United Nations (2023). Japan's aging rate will rise sharply and reach 38.6% in 2065. The figure shows that the population ratio of elderly individuals will rise remarkably compared to other advanced countries, such as Germany, France, the U.S., and the United Kingdom. Thus, Japan's speed and magnitude of demographic aging are remarkable, even compared to other countries that face similar challenges.

Figure 3 shows historical and projected total fertility rates since 1990. The rates first decreased, bottoming at 1.26 in 2005 before somewhat recovering and peaking at 1.45 in 2015. After 2015, fertility rates again decreased, falling to 1.26 in 2022. This figure is the lowest since the start of the Current Population Survey in 1947, indicating an accelerated birthrate decline. The National Institute of Population and Social Security Research (2023) projects that the total fertility rate will recover, reaching 1.33 around 2035 and 1.36 by 2070.

Finally, we consider the effects of the COVID-19 pandemic (which affected the entire world) on an aging and depopulating Japan. The spread of COVID-19 has had severe consequences in the form of accelerated population decline due to a sharp drop in births. Infection control measures included the prevalence of telework, travel, and fewer events; even if people could meet face-to-face, they wore masks and kept their distance (social distance), decisively reducing the opportunities for men and women to meet. Figure 4 shows historical and projected births and actual marriage results since 2010. First, the number of births in Japan was in a critical situation, even before the COVID-19 outbreak, falling significantly below the future population projection (medium projection) by the National Institute of Population and Social Security Research (2017). From there, the number of births in Japan declined significantly, partly because of the spread of COVID-19. The number of births in 2023 fell to 758,631 (preliminary results) from 770,759 in 2022, the lowest since the start of the Current Population Survey in 1899, indicating an accelerated birthrate decline. Moreover, according to the Ministry of Health, Labor and Welfare (2024), the number of marriages (a prerequisite for childbearing in Japan) has decreased sharply. The number of marriages declined to 489,281 couples (preliminary results) in 2023 from 504,930 in 2022, the lowest in the postwar period.

1.2. Public pension in Japan

The current Japanese public pension program is operated fundamentally by a pay-as-you-go (PAYG) style and is two-tiered: a basic flat pension and an amount proportional to each household's average annual labor income. In other words, the pension consists of two parts. The first is a basic pension (*kiso nenkin*), provided for all retirees conditional on paying a required premium. The second is an employment-based part (*kosei nenkin*), whose benefits are based on the premium contribution made by each individual throughout their career. According to the data from the Organisation for Economic Cooperation and Development (OECD) (2023a), total pension expenditures in Japan are 9.3% of the gross domestic product (GDP) in 2022, whereas it is, on average, 7.7% for the 38 OECD member countries. Figure 2 illustrates that Japan's aging rate is expected to rise rapidly compared to other developed countries. Therefore, Japan's GDP ratio of total pension expenditures would be increasingly higher than that of other developed countries.

The Japanese government implemented a significant pension reform in 2004. The purpose of the reform was to make the Japanese public pension system sustainable in the long run. The "macroeconomic slide" was introduced to control Japan's rapid rise in benefit expenditures. The mechanism automatically adjusts benefits downward with a rise in average life expectancy and a decline in insured individuals; however, the macroeconomic slide has been triggered only five times (in 2015, 2019, 2020, 2023, and 2024) since the reform was implemented in 2004 because the adjustment is subject to enough inflation as the downward adjustment is restricted in nominal terms; the adjustment does not exceed the (negative) inflation rate under a deflationary economy. The number of pension recipients in Japan is currently increasing, and the average number of years of receiving public pensions is simultaneously increasing as life expectancy continues to rise. The 2004 pension reform set the timeline in which the contribution rate gradually increased by 0.354 percentage points yearly, from 13.58% of earnings to 18.3% in 2017; since 2017, it has been fixed at 18.3%.

The normal retirement age is defined as the age at which individuals are entitled to receive pension benefits in full, which is set by the policy. In contrast, the age at which individuals exit the labor force depends on an individual's choice. They do not have to coincide and differ for most individuals; however, our simulation results revealed that individuals who maximize their utility work until the compulsory retirement age (from 64 to 69 for our simulation cases). In the benchmark simulation, the normal retirement age is assumed to be 65; individuals work up to 64 and start receiving pension benefits from 65 throughout the entire period. In our model, after households retire at the end of the year in which they reach compulsory retirement, they immediately start receiving full pension benefits from the beginning of the following year. Additionally, our model does not assume accelerated or deferred pension benefits.

The standard starting age for receiving basic and employment-based pensions is currently 65. Individuals could start claiming benefits as early as 60 years old or delaying the initial take-up until as late as 75. Early retirement at a reduced benefit is currently possible at 60 in basic and earnings-related schemes. The benefit is reduced by 0.4% per month of early retirement, i.e., 4.8% per year. Late retirement is currently possible between 65 and 75; deferral increases the pension benefit by 0.7% per month, i.e., 8.4% per year; however, most individuals claim benefits at the normal retirement age (65), and very few people wait until after the normal retirement age to receive benefits. According to Table 26 in the Ministry of Health, Labour, and Welfare (2023b), among those who started to receive national pension benefits in 2022, 87.3% claimed at normal retirement age (65), and 10.8% and 2.0% claimed at ages below and above the normal retirement age, respectively. This report also reveals that those who claimed at ages below the normal retirement age tend to decrease, and those who claimed at the normal retirement age and above tend to increase.

Table 1 compares the standard retirement age and life expectancy for five major developed countries. The average retirement age in Japan is 65 for the basic pension and employment-based part.

Many other countries have (or will have) the normal retirement age of 67 under current regulations, but for Japan, the retirement age remains 65 under current regulations. Table 1 shows that Japan has (or will have) the lowest normal retirement age among the five advanced countries without new regulations. Given the life expectancy across these countries (OECD, 2023a), Japan has the longest expected duration of receiving public pension benefits (19.8 years), approximately 6.1 years longer than the average across the OECD member countries (13.7 years). This report reveals that the expected duration of receiving pension benefits is much longer in Japan than in other major advanced countries; however, the standard starting age for receiving benefits in Japan remains lower than in other countries.

1.3. Employment for elderly individuals in Japan

As Japan's population declines and its average age rises, the diminished working-age cohort faces increasing challenges to support the ballooning numbers of seniors under a PAYG social security system. According to Higo et al. (2016), Japan is far ahead of the rest of the world in terms of the aging population curve and faces unprecedented pressure to delay workers' retirement. In this national context, workers' retirement decisions and behaviors are primarily determined by the interplay between the government and employers over reforming mandatory retirement corporate policies. The policies are thoroughly and uniformly institutionalized across the country's workplaces, set at ages much younger than in most developed countries worldwide. The primary micro-level determinants of retirement include persistent gender roles, which still affect many female workers (who currently aged 50 and older) as they are often primary caregivers for children, aged parents, and the household.

Table 2 presents the employment rate by age groups in 2022 for male–female totals, based on data from the Statistics Bureau of Japan (2023b). The employment rate for the age group from 25 to 59 is high (more than 80%). It declines to 73.0% for the 60–64 age group and 50.8% significantly for the 65–69 age group; however, the employment rate for both age groups has recently tended to increase.[2] Figure 5, based on data from the Ministry of Internal Affairs and Communications (2023a), illustrates the transition of the employment rate for individuals aged 65–69. This figure suggests that the male employment rate is higher than that of females, and both employment rates tend to increase.

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[2] For simplicity, our model assumes that the individual's employment rate until 64 is 100% for all simulation cases. This paper focuses on the analysis of rises in the standard starting age for receiving pension benefits under different employment rates for individuals aged 65 and above; therefore, the assumption of the 100% employment rate until 64 would not have essential effects on our main findings.

The remainder of this paper is organized as follows. Section 2 describes literature related to this study, Section 3 identifies the basic model applied in the simulation analysis, Section 4 explains the method and assumptions of simulation analysis, Section 5 evaluates the simulation findings, and Section 6 summarizes, concludes, and discusses policy implications.

2. Related Literature

This paper contributes to the literature on public pension reform in an aging society, especially in Japan. We focus on five previous studies that analyzed Japan's public pension issue; the primary literature is as follows.

First, we discuss two papers that analyzed the public pension reform—Kitao (2015) and Kitao (2017)—which are the most important for our study.

Kitao (2015) quantified the fiscal cost of the demographic transition that Japan is projected to experience over the next several decades. The author used a lifecycle model with endogenous saving, consumption, and labor supply in both intensive and extensive margins. Retirement waves of baby-boom generations, combined with a rise in longevity and low fertility rates, raised the old-age dependency ratio to 85% by 2050, the highest among major developed countries. A significant budget imbalance occurred as the government faced rising public pension and health and long-term care insurance costs. Preserving the current level of the transfers would require a significant increase in taxation. Kitao (2015) found that the tax rate reached the maximal value of 48% in the late 2070s, using consumption taxes to balance the government budget. A pension reform to reduce benefits by 20% resulted in a peak tax rate of 37%, which could be reduced further to 28% if the retirement age was also gradually raised by 5 years.

Kitao (2017) simulated pension reform to reduce the replacement rate by 20% and gradually raise the retirement age by 3 years over 30 years. That author considered three scenarios with different points in time to initiate reform in 2020, 2030, and 2040, respectively, finding that a delay would suppress economic activities, lowering output by up to 4% and raising the tax burden by more than 8% of total consumption. Delaying reform implied a transfer of costs of demographic aging to the young and thus deteriorated the welfare of future generations by up to 3% in terms of consumption equivalence.

Both papers considered the reduction of pension benefits by 20% and the rise of the normal retirement age as pension reform measurements in Japan. In Kitao (2015), the standard retirement age was raised from 65 to 67 and 70, whereas in Kitao (2017), it gradually rose from 65 to 68 over 30 years.

Our study considers raising the normal retirement age from 65 to 68 and 70, but the transitional period for the changes is not as long as the assumption in those papers (explained in Subsection 4.2).

We next examine three previous studies related to public pension reforms in Japan. Okamoto (2013) used a computable general equilibrium model with overlapping generations to explore the effects of different public pension schemes on economic welfare and intergenerational and intragenerational equity. Besides the benchmark case based on the 2004 public pension reform, the author considered two alternative reforms: financing the basic pension benefit through a consumption tax and eliminating the earnings-related pension benefit. The study's results suggested that even the consumption-tax financing of only the basic pension (i.e., the combination of both reforms) might not improve overall economic welfare, although it increases economic output by inducing capital formation.

Braun and Joines (2015) modeled details of medical expenditures and the health insurance system, examining the effects of public pension and health insurance reform in a general equilibrium lifecycle model. They found that Japan's aging population was already burdening government finances and that the very high debt-GDP ratio constrains its ability to confront the negative fiscal implications of future aging. They also found that Japan faces a severe fiscal crisis without imminent remedial action and analyzed alternative strategies for correcting Japan's fiscal imbalances.

Maebayashi (2019) investigated how unfunded public pensions financed by value-added tax (VAT), as discussed in Japan, affect economic growth, examining whether payroll tax (PT) or VAT is the more growth-friendly tax structure for financing public pensions. The author studied these issues using overlapping generations models with parental altruism, determining that a public pension system financed by VAT may increase economic growth when bequests are operative. In contrast, public pensions hinder growth when bequests are inoperative unless agents are sufficiently patient. The results also indicated that public pensions financed by VAT are more growth-friendly than those financed by PT.

3. Theoretical Framework

We calibrate the simulation of the Japanese economy by applying population data from 2023, estimated by the National Institute of Population and Social Security Research. The model includes 106 overlapping generations, corresponding to ages 0–105 years old. Three types of agents are incorporated: households, firms, and the government. The following subsections describe the basic structures of households, firms, and the government, as well as the market equilibrium conditions.

Our model incorporates intergenerational mobility across income classes based on Kikkawa (2009) who found that Japan's income disparity stems fundamentally from different educational backgrounds between high school and university graduates. On the basis of his study, our model introduces two types of representative agents: the low-income class (i.e., (just) high school graduates) and the high-income class (i.e., university graduates) into a cohort. In this section, we describe the behavior of the low-income class household in the model (see Appendix A for the behavior of the high-income class).

3.1. Household behavior

The economy is populated by 106 overlapping generations that live with uncertainty, corresponding to ages 0–105. Each agent is assumed to consist of a neutral individual because our model does not distinguish by gender. Each agent enters the economy as a decision-making unit and starts to work at age 18 years, and lives to a maximum age of 105 years. Each household is assumed to consist of one adult and its children. The children aged 0–17 or 0–21 only consume, involving childrearing costs for their parent. Each household faces an age-dependent probability of death. Let $q_{j+1|j}^{t}$ be the conditional probability that a household born in year t lives from age j to j +1. Then the probability of a household born in year t, surviving until s can be expressed by

$$p_s^{t(H)} = \prod_{j=18}^{s-1} q_{j+1|j}^{t}. \quad (1)$$

The probability $q_{j+1|j}^{t}$ is calculated from data estimated by the National Institute of Population and Social Security Research (2023). Since the survival probability is different among agents with different birth year, agents born in different years have the different utility function.

Each agent who begins its economic life at age 18 chooses perfect-foresight consumption paths (C_s^t), leisure paths (l_s^t), and the number of born children (n_s^t) to maximize a time-separable utility function of the form:

$$U^{t(H)} = \frac{1}{1-\frac{1}{\gamma}}\left[\alpha^{(H)} \sum_{s=18}^{40} p_s^{t(H)} (1+\delta)^{-(s-18)} \left(n_s^{t(H)}\right)^{1-\frac{1}{\gamma}} + (1 - \alpha^{(H)}) \sum_{s=18}^{105} p_s^{t(H)} (1+\delta)^{-(s-18)} \left\{\left(C_s^{t(H)}\right)^{\varphi} \left(l_s^{t(H)}\right)^{1-\varphi}\right\}^{1-\frac{1}{\gamma}}\right]. \quad (2)$$

This utility function represents the lifetime utility of the agent born in year t. $C_s^{t(H)}$, $l_s^{t(H)}$ and $n_s^{t(H)}$ are respectively consumption, leisure and the number of children to bear (only in the first 23 periods of the life) for an agent born in year t, of age s; $\alpha^{(H)}$ is the utility weight of the number of children relative to the consumption–leisure composite, γ is the intertemporal elasticity of substitution, δ is the adjustment coefficient for discounting the future, and φ is the consumption share parameter to leisure.

Fertility choice in the model is only based on the direct utility that households obtain from their offspring, neglecting the investment element of children. The demand for children as *investment goods* played an important role in traditional economies (and still does in developing countries), where transfers from the young to the old arise within the family. In modern advanced countries, however, a PAYG social security scheme makes the investment aspect of children socialized, as Groezen *et al.* (2003) pointed out. This creates the possibility for households to free-ride on the scheme by rearing fewer or no children, still being entitled to a full pension benefit. Therefore, we treat children as "consumption goods" and a parent is assumed to obtain the utility from the number of children born at each age.

Letting $A_s^{t(H)}$ be capital holdings for the agent born in year t, of age s, maximization of Equation (2) is subject to a lifetime budget constraint defined by the sequence:

$$A_{s+1}^{t(H)} = \{1 + r_{t+s}(1 - \tau^r)\}A_s^{t(H)} + (1 - \tau^w - \tau_{t+s}^p)w_{t+s}e_s^{(H)}\{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\}\Lambda_s + a_s^{t(H)} - or_s^{t(H)} + b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) - (1 + \tau_{t+s}^c)C_s^{t(H)} - (1 - m)(1 + \tau_{t+s}^c)\Phi_s^{t(H)} - m(1 + \tau_{t+s}^c)\Phi_s^{t(U)}, \quad (3)$$

where r_t is the pretax return to savings, and w_t is the real wage at time t; τ^w, τ^r and τ_t^c are the tax rates on labor income, capital income and consumption, respectively. τ_t^p is the contribution rate to the public pension scheme at time t. All taxes and contributions are collected at the household level. $tc(n^{(H)})$ is the time cost for childrearing. $a^{(H)}$ is the bequest to be inherited, and $or^{(H)}$ is the childrearing cost for orphans. There are no liquidity constraints, and thus the assets $A_s^{(H)}$ can be negative. Terminal wealth must be zero. An individual's earnings ability $e_s^{(H)}$ is an exogenous function of age, and Λ_s denotes the employment rate of age s.

The public pension program is assumed to be a PAYG scheme similar to the current Japanese system. The program starts to collect contributions to the scheme from the age of 20, in accordance with the law. The pension benefit is assumed to comprise only an earnings-related pension:

$$b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) = \begin{cases} \theta H^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) & (s \geq ST) \\ 0 & (s < ST) \end{cases}, \quad (4)$$

where

$$H^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)=\frac{1}{RE-19}\sum_{s=20}^{RE}w_{t+s}e_s^{(H)}\{1-l_s^{t(H)}-tc_s^t(n_u^{t(H)})\}. \quad (5)$$

The age at which a household born in year t starts to receive the public pension benefit is ST, the average annual labor income for the calculation of pension benefit for each agent is $H^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$, and the weight coefficient of the part proportional to $H^{t(H)}$ is θ. The symbol $b_s^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$ signifies that the amount of public pension benefit is a function of the age profile of labor supply, $\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}$.

A parent is assumed to bear children with the upper limit of 40 years old, and expend for them until they become independent of their parent, namely, during the period when children are from zero to 17 or 21 years old. Regarding the childrearing costs, the model takes account of both monetary and time costs. Here note that the children aged below 18 or 22 years old do not conduct an economic activity independently, and childrearing costs for their parent arise until they become independent of their parent. The financial costs for rearing the children, for the parent born in year t and s years old, are represented by $\Phi_s^{t(H)}$ and $\Phi_s^{t(U)}$, which are the cost for the children who will become high school graduates and university graduates, respectively:

$$\Phi_s^{t(H)}=\begin{cases}\sum_{k=18}^{s}\xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=18,19,\cdots,35)\\ \sum_{k=s-17}^{s}\xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=36,37,\cdots,40),\\ \sum_{k=s-17}^{40}\xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=41,42,\cdots,57)\end{cases} \quad (6)$$

$$\Phi_s^{t(H)}=0\ \ (s=58,59,\cdots,105), \quad (7)$$

$$\Phi_s^{t(U)}=\begin{cases}\sum_{k=18}^{s}\xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=18,19,\cdots,39)\\ \sum_{k=s-21}^{40}\xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=40,41,\cdots,61)\end{cases}, \quad (8)$$

$$\Phi_s^{t(U)}=0\ \ (s=62,63,\cdots,105), \quad (9)$$

$$\xi^{t(H)}=\beta NW^{t(H)}, \quad (10)$$

where $\xi^{t(H)}$ is the childrearing cost for the parent born in year t, ρ is the rate of government subsidy (including child allowances) to childrearing costs, and β is the ratio of childrearing costs to the net lifetime income, $NW^{t(H)}$, for the parent born in year t.

The children who will become university graduates needs more monetary cost than the children who will become high school graduates simply by the extra four-year (18–21) cost before the independence from their parents. The mobility m denotes the probability in which the children will belong to the high-income class (i.e., university graduates) different from their parent, and $1-m$ is the probability in which they will belong to the low-income class (i.e., high school graduates) same as their parent. The number of children affects the whole available time for a parent, because of the time required for childrearing. The time cost for rearing the children for the parent born in year t, of age s, is represented by

$$tc_s^{t(H)} = \mu n_s^{t(H)}, \tag{11}$$

where μ is the parameter that shows the relation between the number of children and the time required for childrearing, which is simply assumed to be proportional to the number of born children. The time cost is assumed to be same across the two types of children who will become high school graduates or university graduates.

The model contains accidental bequests that result from uncertainty over length of life. The bequests, which comprise assets previously held by deceased households, are distributed equally among all surviving low-income class households at time t. When $BQ_t^{(H)}$ is the sum of bequests inherited by the low-income class households at time t, the bequest to be inherited by each low-income household is defined by

$$a_s^{t(H)} = \frac{(1-\tau^h)BQ_{t+s}^{(H)}}{E_{t+s}^{(H)}}, \tag{12}$$

where

$$BQ_t^{(H)} = \sum_{s=18}^{105}(N_s^{t-s-1(H)} - N_{s+1}^{t-s-1(H)})A_{s+1}^{t-s-1(H)}. \tag{13}$$

τ^h is the tax rate on inheritances of bequests. The amount of inheritances received is linked to the age profile of assets for each household. $E_t^{(H)}$ is the number of the low-income class households conducting an economic activity independently, aged 18 and older. The number of the generation with age s years born in year t is represented by

$$N_s^{t(H)} = p_s^{t(H)} N_0^{t(H)}. \tag{14}$$

Total childrearing cost of the orphans, who are generated as a consequence of parents' uncertainty over length of life, is distributed equally among all surviving low-income class households at time t. When $OR_t^{(H)}$ is the sum of childrearing costs incurred by the low-income class households at time t, the childrearing cost for orphans for each low-income class household is defined by

$$or_s^{t(H)} = \frac{OR_{t+s}^{(H)}}{E_{t+s}^{(H)}}, \tag{15}$$

where

$$OR_t^{(H)} = (1-m)\sum_{s=18}^{57}(N_{s-1}^{t-s(H)} - N_s^{t-s(H)})\Phi_s^{t-s(H)} + m\sum_{s=18}^{61}(N_{s-1}^{t-s(H)} - N_s^{t-s(H)})\Phi_s^{t-s(U)}. \tag{16}$$

Therefore, the net amount of bequests is represented as $a^{(H)} - or^{(H)}$. When we consider the utility maximization problem over time for each agent, besides the flow budget constraint represented by Equation (3), the following constraint is imposed:

$$\begin{cases} 0 \le l_s^{t(H)} \le 1 - tc_s^t(n_s^{t(H)})(18 \le s \le RE) \\ \quad l_s^{t(H)} = 1\ (RE + 1 \le s \le 105) \end{cases}. \tag{17}$$

This is a constraint that labor supply is nonnegative, and that each household inevitably retires after passing the compulsory retirement age, RE.

Let us consider the case where each agent maximizes expected lifetime utility under two constraints. Each individual maximizes Equation (2) subject to Equations (3) and (17) (see Appendix B for further details). From the utility maximization problem, the equation expressing the evolution of the number of children over time for each individual is characterized by

$$W_s^{t(H)} = \left(\frac{p_{s-1}^{t(H)}}{p_s^{t(H)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right] W_{s-1}^{t(H)}, \tag{18}$$

$$W_s^{t(H)} = \frac{\alpha^{(H)} k^{1-\frac{1}{\gamma}} (n_s^{t(H)})^{-\frac{1}{\gamma}}}{(1+\tau_{t+s}^c)\left[(1-m)\sum_{g=0}^{17} \Omega_{s,g}^t \xi^{t(H)}(1-\rho) + m\sum_{g=0}^{21} \Omega_{s,g}^t \xi^{t(H)}(1-\rho)\right]}, \tag{19}$$

where $\Omega_{s,0}^t$=1 for $g = 0$, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g}\{1 + r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$.

Similarly, that for the consumption–leisure composite is represented by

$$V_s^{t(H)} = \left(\frac{p_{s-1}^{t(H)}}{p_s^{t(H)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right] V_{s-1}^{t(H)}, \tag{20}$$

$$V_s^{t(H)} = \frac{(1-\alpha^{(H)})\left\{(C_s^{t(H)})^{\varphi}(l_s^{t(H)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}} \varphi (C_s^{t(H)})^{\varphi-1}(l_s^{t(H)})^{1-\varphi}}{1+\tau_t^c}. \tag{21}$$

3.2. Firm behavior

The model has a single production sector that is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. Capital is homogeneous and depreciating, while labor differs only in efficiency. All forms of labor are perfectly substitutable. Households with different income classes or different ages, however, supply different amounts of some standard measure per unit of labor input.

The aggregate production technology is the standard Cobb-Douglas form:

$$Y_t = K_t^{\varepsilon} L_t^{1-\varepsilon}, \tag{22}$$

where Y_t is aggregate output (national income), K_t is aggregate capital, L_t is aggregate labor supply measured by the efficiency units, and ε is capital's share in production. Using the property subject to a constant-returns-to-scale production function, we can obtain the following equation:

$$Y_t = (r_t + \delta^k) K_t + w_t L_t, \tag{23}$$

where δ^k is the depreciation rate.

3.3. Government behavior

At each time t, the government collects tax revenues and issues debt (D_{t+1}) that it uses to finance government purchases of goods and services (G_t) and interest payments on the inherited stock of debt (D_t). The government sector consists of a narrow government sector and a pension sector, and a portion of revenues is transferred to the public pension sector. The public pension system is assumed to be a simple PAYG style and consists only of earnings-related pension. Pension account expenditure is financed by both contributions and a transfer from the general account.

The budget constraint of the narrower government sector at time t is given by

$$D_{t+1} = (1 + r_t)D_t + G_t - T_t, \tag{24}$$

where G_t is total government spending on goods and services, T_t is total tax revenue from labor income, capital income, consumption and inheritances, and D_t is the net government debt at the beginning of year t. D_t is gross public debt minus the accumulated pension fund because the model abstracts the public pension fund, which is represented as a ratio to national income:

$$D_t = dY_t, \tag{25}$$

where d is the ratio of net public debt to national income.

The public pension system is assumed to be a simple PAYG style. The budget constraint of pension sector at time t is represented by

$$R_t = (1 - \pi)B_t, \tag{26}$$

where R_t is total revenue from contributions to the pension program, B_t is total spending on the pension benefit to generations of age ST and above, and π is the ratio of the part financed by the tax transfer from the general account.

The total government spending on goods and service is defined by

$$G_t = gY_t + \pi B_t + GS_t, \tag{27}$$

where G_t includes transfers to the public pension sector (πB_t) and the government subsidies to child rearing (GS_t). The government spending except for the transfers and the subsidies is gY_t, which is assumed to be represented as a constant ratio (g) of national income. The spending is assumed to either generate no utility to households or enter household utility functions in a separable fashion.

The total amount of government subsidies (including child allowances) to the childrearing cost in year t is GS_t:

$$GS_t = GS_t^{(H)} + GS_s^{(U)}, \tag{28}$$

$$GS_t^{(H)} = \rho\left[(1-m)\sum_{s=18}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)} + RC_{s,t}^{c(H)}) + m\sum_{s=18}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)})\right], \tag{29}$$

$$\begin{cases} RC_{s,t}^{a(H)} = \sum_{k=18}^{s} N_k^{t-s(H)} \xi^{t-s(H)} n_k^{t-s(H)} & (s = 18,19,\cdots,35) \\ RC_{s,t}^{b(H)} = \sum_{k=s-17}^{s} N_k^{t-s(H)} \xi^{t-s(H)} n_k^{t-s(H)} & (s = 36,38,\cdots,40), \\ RC_{s,t}^{c(H)} = \sum_{k=s-17}^{40} N_k^{t-s(H)} \xi^{t-s(H)} n_k^{t-s(H)} & (s = 41,42,\cdots,57) \end{cases} \tag{30}$$

$$\begin{cases} RC_{s,t}^{a(U)} = \sum_{k=18}^{s} N_k^{t-s(H)} \xi^{t-s(H)} n_k^{t-s(H)} & (s = 18,19,\cdots,39) \\ RC_{s,t}^{b(U)} = \sum_{k=s-21}^{40} N_k^{t-s(H)} \xi^{t-s(H)} n_k^{t-s(H)} & (s = 40,41,\cdots,61) \end{cases}, \tag{31}$$

where $RC_t^{a(H)}$, $RC_t^{b(H)}$ and $RC_t^{c(H)}$ are monetary costs for childrearing when the children will belong to the low-income class same as their parent, namely, they will become high school graduates, and $RC_t^{a(U)}$ and $RC_t^{b(U)}$ are the costs when the children will belong to the high income class different from their parent, namely, they will become university graduates.

$$GS_t^{(U)} = \rho\left[(1-m)\sum_{s=22}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)} + RC_{s,t}^{c(U)}) + m\sum_{s=22}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)})\right], \tag{29'}$$

$$\begin{cases} RC_{s,t}^{a(U)} = \sum_{k=22}^{s} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 22,23,\cdots,40) \\ RC_{s,t}^{b(U)} = \sum_{k=22}^{40} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 41,42,43) \\ RC_{s,t}^{c(U)} = \sum_{k=s-21}^{40} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 44,45,\cdots,61) \end{cases}, \tag{30'}$$

$$\begin{cases} RC_{s,t}^{a(H)} = \sum_{k=22}^{s} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 22,23,\cdots,39) \\ RC_{s,t}^{b(H)} = \sum_{k=s-17}^{40} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 40,41,\cdots,57) \end{cases}, \tag{31'}$$

where $RC_t^{a(U)}$, $RC_t^{b(U)}$ and $RC_t^{c(U)}$ are financial costs for childrearing when the parent is 22 to 61 years old. Once the parent becomes 62 years old, the cost does not exist because all children are independent from their parent.

The total spending on the pension benefit to generations of age ST and above is represented by

$$B_t = B_t^{(H)} + B_t^{(U)}, \tag{32}$$

where $B_t^{(H)}$ and $B_t^{(U)}$ are the expenditure for the two income classes:

$$B_t^{(H)} = \sum_{s=ST}^{105} N_s^{t-s(H)} b_s^{t-s(H)}, \tag{33}$$

$$B_t^{(U)} = \sum_{s=ST}^{105} N_s^{t-s(U)} b_s^{t-s(U)}. \tag{33'}$$

The total revenue from pension contributions and the total tax revenue are represented by

$$R_t = \tau^p w_t L_t, \tag{34}$$

$$T_t = \tau^w w_t L_t + \tau^r r_t AS_t + \tau_t^c AC_t + \tau^h BQ_t, \quad (35)$$

where aggregate assets supplied by households, AS_t, and aggregate consumption, AC_t, are given by

$$AS_t = AS_t^{(H)} + AS_t^{(U)}, \quad (36)$$

$$AC_t = AC_t^{(H)} + AC_t^{(U)}. \quad (37)$$

For the low-income class, aggregate assets supplied by households, $AS_t^{(H)}$, and aggregate consumption, $AC_t^{(H)}$, are given by

$$AS_t^{(H)} = \sum_{s=18}^{105} N_s^{t-s(H)} A_s^{t-s(H)}, \quad (38)$$

$$AC_t^{(H)} = \sum_{s=18}^{105} N_s^{t-s(H)} C_s^{t-s(H)} + (1-m)\sum_{s=18}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)} + RC_{s,t}^{c(H)}) + m\sum_{s=18}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)}), \quad (39)$$

where aggregate consumption consists of adult's consumption (at age 18–105 years old) and children's consumption or cost (at age zero to 17 or 21 years old).

For the high-income class, aggregate assets supplied by households, $AS_t^{(U)}$, and aggregate consumption, $AC_t^{(U)}$, are given by

$$AS_t^{(U)} = \sum_{s=22}^{105} N_s^{t-s(U)} A_s^{t-s(U)}, \quad (38')$$

$$AC_t^{(U)} = \sum_{s=22}^{105} N_s^{t-s(U)} C_s^{t-s(U)} + (1-m)\sum_{s=22}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)} + RC_{s,t}^{c(U)}) + m\sum_{s=22}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)}), \quad (39')$$

where aggregate consumption consists of adult's consumption (at age 22–105 years old) and children's consumption or cost (at age zero to 21 or 17 years old).

The total sum of bequests inherited by the households and the total childrearing cost of the orphans at time t are as follows:

$$BQ_t = BQ_t^{(H)} + BQ_t^{(U)}, \quad (40)$$

$$OR_t = OR_t^{(H)} + OR_t^{(U)}. \quad (41)$$

Total population (i.e., the population aged zero to 105), the population aged 18 or 22 to 105 (i.e., independents financially), and the population aged 65 to 105 (i.e., retirees) in year t are respectively represented by

$$Z_t = Z_t^{(H)} + Z_t^{(U)}, \quad (42)$$

$$E_t = E_t^{(H)} + E_t^{(U)}, \quad (43)$$

$$O_t = O_t^{(H)} + O_t^{(U)}. \quad (44)$$

The aging rate (i.e., the old-age dependency ratio), the ratio of the population aged 65 and above to the total population, is given by O_t/Z_t. For the low-income class, the total population, the population aged 18 to 105, and the population aged 65 to 105 in year t are respectively represented by

$$Z_t^{(H)} = \sum_{k=0}^{105} \sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}, \quad (45)$$

$$E_t^{(H)} = \sum_{k=18}^{105} \sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}, \quad (46)$$

$$O_t^{(H)} = \sum_{k=65}^{105} \sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}. \quad (47)$$

For the high-income class, the total population, the population aged 22 to 105, and the population aged 65 to 105 in year t are respectively represented by

$$Z_t^{(U)} = \sum_{k=0}^{105} \sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}, \quad (45')$$

$$E_t^{(U)} = \sum_{k=22}^{105} \sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}, \quad (46')$$

$$O_t^{(U)} = \sum_{k=65}^{105} \sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}. \quad (47')$$

3.4. Market equilibrium

Finally, equilibrium conditions for the capital, labor and goods markets are described.

1) Equilibrium condition for the capital market

Because aggregate assets supplied by households equal the sum of real capital and net government debt,

$$AS_t = K_t + D_t. \quad (48)$$

2) Equilibrium condition for the labor market

Measured in efficiency units, because aggregate labor demand by firms equals aggregate labor supply by households,

$L_t = L_t^{(H)} + L_t^{(U)}$, (49)

where $L_t^{(H)} = \sum_{s=18}^{RE} N_s^{t-s(H)} e_s^{(H)} \{1 - l_s^{t-s(H)} - tc_s^t(n_s^{t(H)})\}$, (50)

$L_t^{(U)} = \sum_{s=22}^{RE} N_s^{t-s(U)} e_s^{(U)} \{1 - l_s^{t-s(U)} - tc_s^t(n_s^{t(U)})\}$. (50')

3) Equilibrium condition for the goods market

Because aggregate production equals the sum of private consumption, private investment and government expenditure,

$$Y_t = AC_t + \{K_{t+1} - (1 - \delta^k) K_t\} + G_t. \quad (51)$$

An iterative program is performed to obtain the equilibrium values of the above equations.

4. Simulation Analysis

4.1. Method

The simulation model presented in the previous section is solved, given the assumption that households have fundamentally perfect foresight and correctly anticipate interest, wages, the tax and contribution rates, and other factors such as the normal retirement age and the employment rate for the elderly. If the tax and social security systems and other elements are determined, then the model can be solved using the Gauss–Seidel method (see Auerbach and Kotlikoff (1987) and Heer and Maußner (2005) for the computation process).

Our study assumes the transitional economy of Japan from the initial steady state in 2022 to the final steady state in 2300. Alternative scenarios with the different normal retirement age and the different employment rate for the elderly are assumed to be implemented at the end of 2022. For simplicity, 2022 is set as the starting year, and we simulate the demography and the economy in the following years. For the generations that were alive in 2022 and have survived in 2023, we need to pay attention to their formation of future expectations. In 2023, these generations realized that their previous expectations no longer apply and thus again maximize their remaining lifetime utility given perfect foresight. Based on the ex-post age profiles of the number of children to bear, consumption, and leisure for these generations, we calculated their lifetime utility at 18 and 22 years for the low- and high-income classes, respectively.

The LSRA first transfers to each household affected by the reform just enough resources (possibly a negative amount) to return its expected remaining lifetime utility to its pre-change level in the benchmark simulation. For each household that is alive when a change occurs at the end of 2022, at its age in 2023, the LSRA makes a lump sum transfer, to return its expected remaining lifetime utility to its pre-change utility level. The LSRA also makes a lump-sum transfer to each future household that enters the economy after a change (from 2023 onward), at its age of 18 or 22 years, to return its expected entire lifetime utility back to its pre-change level.

Note that the net present value of these transfers in 2023 across living and future households will generally not sum to 0. Thus, the LSRA makes an additional lump sum transfer to each future household so that the net present value across all transfers is 0. To illustrate, let us assume that these additional transfers are uniform across all future generations, including the low- and high-income classes. If the transfer is positive, then the change has produced extra resources after the expected remaining lifetime utility of each household has been restored to its pre-change level. In this case, we can interpret that the change has created efficiency gains, i.e., *Pareto improvements*. Conversely, if the transfer is negative, then the change has generated an efficiency loss. Thus, the total net present value of all lump sum transfers to current and future generations sums to 0 in 2023, satisfying the LSRA budget constraint (see Nishiyama and Smetters (2005) for further details).

4.2. Simulation cases

This study uses an extended lifecycle general equilibrium model with endogenous fertility to investigate the quantitative effects of a rise of the normal starting age for receiving pension benefits in Japan on individual welfare and future demographics. We assume the starting age for pension benefits is 65 in the 2022 initial steady state. Table 3 presents the simulation cases considered in our analysis. The benchmark simulation assumes that the starting age remains 65 throughout the entire period. We consider two scenarios for the six alternative cases where the starting ages for receiving pension benefits rise to 68 and 70. For each scenario, we also consider three experiments in which the employment rates for individuals aged 65 and above are 50%, 75%, and 100%, respectively.

We assume that the changes are gradually performed to avoid extra disturbance or confusion from sudden rises in the standard starting age for receiving pension benefits. The starting age is 66 for people born in 1958, 67 in 1959, 68 in 1960, 69 in 1961, and 70 in 1962. For the scenarios with the 68-pension starting age, it is 68 for the people born after 1960. For the scenarios with the 70-pension starting age, it is 70 for the people born after 1962. Additionally, we will consider the case with LSRA transfers for each scenario. To distinguish potential efficiency gains/losses from possibly offsetting changes in the welfare of different generations, we introduce LSRA into the alternative six scenarios, Cases 68-50%, 68-75%, 68-100%, 70-50%, 70-75%, and 70-100% in Table 3. The LSRA transfers produce a leveled and common welfare gain/loss for each future household, including the low- and high-income classes.

4.3. Specification of the parameters

We chose realistic parameter values for the Japanese economy based on the literature (Nishiyama and Smetters, 2005; Oguro et al., 2011; İmrohoroğlu et al., 2017; Kitao and Mikoshiba, 2020). Table 4 displays the parameter values assigned in the baseline simulation, and the data source used in the calibration. Parameter values were chosen such that the calculated values of the model's endogenous variables approached the actual data values. Table 5 presents the endogenous variables in the 2022 initial steady state. Because the simulation results depend on the model setting and the given parameters, we must be careful about the effects of any parameter changes.

4.3.1. Demography

Japan's population is aging at an unprecedented speed for a developed nation; simultaneously, the population is decreasing, which has become one of Japan's most important problems. Japan's speed and magnitude of demographic aging are remarkable, even compared to other advanced countries facing similar challenges. Our extended lifecycle general equilibrium simulation model with endogenous fertility rigorously reflects such demographic dynamics in Japan.

The age-specific survival probability $q^{t}_{j+1|j}$ is calculated from data estimated by the National Institute of Population and Social Security Research (2023). We used the average values for males and females on future life tables by age from 2020 until the last year for which official projections are available, 2070; after 2070, we used the 2070 life table data. For simplicity, the survival rate for the low-income class (i.e., high school graduates) is unity at 18 years old, and that for the high-income class (i.e., university graduates) is unity at 22.

Table 6 indicates the population ratio of individuals with different educational backgrounds in 2022, estimated from the Basic Survey on Wage Structure (Chingin Sensasu) by the Ministry of Health, Labour, and Welfare (2023a). The population share of high school graduates (including junior high school graduates) and university graduates (including technical and junior college graduates) is 49.4% and 50.6%, respectively.

Figure 6 illustrates the age–population distribution in 2022 based on data from the Statistics Bureau of Japan (2023a), denoting the population of high school and university graduates, respectively, for each age. We estimated each population of high school graduates and university graduates aged 0–105 in 2022, similar to Okamoto (2022, 2024). For the elderly, especially those of advanced age, the number of high school graduates exceeds that of the university graduates, whereas for the young and the middle-aged, it is approximately fifty–fifty. For those who are under 18 or 22 years old and undecided to become high school or university graduates, we assume that their population is the same i.e., fifty–fifty on the basis of Kikkawa (2009).

4.3.2. Preference parameter on the number of children

Regarding the preference parameter for children in the utility function of households, the parameter value in the benchmark simulation is the same between the low-income class (i.e., high school graduates) and the high-income class (i.e., university graduates). In other words, the utility weight of the number of children relative to the consumption–leisure composite in Equations (2) and (2)' is the same between the two income classes ($\alpha^{(H)} = \alpha^{(U)} = 0.028148$). This parameter setting is implemented after comprehensively considering several empirical studies, such as Kikkawa (2018) and Adsera (2017). (2022).

Initially, Kikkawa (2018) suggested that the low-income class tends to have more children than the high-income class. That study presents the scheduled number of children for young people aged 21 to 40, which is based on a large-scale questionnaire survey (SSM2015). Accordingly, on average, the scheduled number of children for young high school-graduate couples is 1.14, whereas it is 0.875 for young university-graduate couples. The data revealed that the low-income class has more children than the high-income class.

Conversely, some previous studies, such as Adsera (2017), revealed that such tendencies have weakened recently. Adsera (2017) investigated the effects of a possible increase in the employment and income gaps between highly educated and low-educated workers on their fertility. They suggested that educational attainment's negative fertility gradient has recently weakened in developed countries, and the gap in the number of children born between more-educated and less-educated women has shrunk. Their study also suggested that rising inequality is one mechanism that could underlie this apparent fertility convergence. As some middle-income jobs seem to disappear, polarization in the labor market has increased. This change in the labor market could exert downward pressure on the fertility of medium- and less-educated couples and further flatten the educational gradient.

The empirical data derived by Kikkawa (2018) show that the fertility rate of the low-income class is higher than that of the high-income class, which is based on a reasonable rationale and has a certain validity; however, the findings of Adsera (2017) suggest that the fertility rate differences between the two income classes have recently decreased. Based on the above considerations, this paper's benchmark assumed that the parameter (α) related to the preference for the number of children in the households' utility function was set to the same value between the two income classes.

The parameter value determining the fertility was chosen so that the total fertility rate (TFR) is 1.26 in the 2022 initial steady state, reflecting that Japan's actual TFR was 1.26 in 2022. Consequently, the parameter value ($\alpha^{(H)}$, $\alpha^{(U)}$) is set to 0.028148; In the initial steady state, the TFR is 1.33 for the low-income class and 1.15 for the high-income class, indicating that the TFR is higher for the low-income class. A possible reason is that the utility obtained from the number of children is relatively higher for the low-income class than the high-income class because of the lower wage income per unit of labor.

4.3.3. Childrearing costs

Next, we describe how we assign parameter values for childrearing since our simulation model incorporates endogenous fertility. Based on empirical data, such as Kikkawa (2009), in our model, 70% of children from the high-income class will become high-income class households, and 70% of children from the low-income class will become low-income. In Japan, the high-income class spends more on educating their children than the low-income class because private education has a higher weight. This fact justifies the model setting that childrearing costs are proportional to the parent's lifetime income.

The Cabinet Office (2010) indicated the average annual childrearing costs for the first-born child to annual income for each age. Based on the survey in the Cabinet Office (2010), we assigned the parameter value of β (i.e., the ratio of childrearing costs to parental net lifetime income) such that the ratio of the annual net childrearing costs to annual labor income for the individual is, on average, close to 19.3%. Thus, β is assigned 0.0385 (the ratio is 19.2 % for the low-income class and 19.1% for the high-income class).

The OECD (2023b) presents public spending on family benefits in cash, services, and tax breaks for families as a percentage of GDP in 2019. For Japan, public spending ratios on family benefits in cash, services, and tax measures to GDP are 0.66%, 1.08%, and 0.20%, respectively.[3] We assigned the value of parameter ρ (government childcare subsidies divided by childrearing cost) to 0.1 in the model, as in Oguro et al. (2011). Consequently, the ratio of total government subsidies to national income was 1.11 % in the 2022 initial steady state.

Additionally, our model incorporated not only the monetary costs of childrearing but also the time costs. Increases in the number of children diminish the parent's available time, because of the time required for childrearing; more children to bear, more time required for childrearing. The parameter determining this relation, μ, is assigned under the simple assumption that one child required 1 h per day for childrearing.[4]

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[3] In Japan, the ratio of total family benefits to GDP is only 1.95%, whereas it is, on average, 2.29% for the 38 OECD member countries. This shows that the level of governmental support for childrearing is considerably lower in Japan than that in other countries.

[4] Calibrating the value of parameter, μ, that determines the time cost in the model is difficult. In the 2022 initial

4.3.4. Age profile of labor efficiency

The age profiles of earning ability for the two income classes were estimated with data from the Basic Survey on Wage Structure (Chingin Sensasu) by the Ministry of Health, Labour and Welfare (2014–2023a) for the 2013–2022 period. Figure 7 illustrates age–earnings profiles by education. The labor efficiency profiles are constructed from the Japanese data on employment, wages, and monthly work hours.

To estimate the age profiles of earnings ability, $e_s{}^{(H)}$ and $e_s{}^{(U)}$, respectively, the following equation is constructed:

$$Q_t = a_0 + a_1 A_t + a_2 A_t^2, \quad (52)$$

where Q is the average monthly cash earnings for high school-graduate workers and university-graduate workers, respectively, and A is the average age for each of the workers, including both males and females. Because bonuses account for a large part of earnings in Japan, Q includes bonuses. Using the above data, we use the ordinary least squares (OLS) method to perform estimation. Figure 7 presents the results, illustrating age–earnings profiles by educational background. For the high school graduates, they start to work earlier (18 years old), but their age profile of earnings is flatter with a lower level than the university graduates. For the university graduates, they start to work later (22 years old), but their age profile of earnings is steeper with a higher level.

Figure 7 shows age–earnings profiles for two representative agents (high school graduates and university graduates) with different employment rates (50%, 75%, and 100%) for individuals aged 65 and above. For Cases 68-100% and 70-100%, the estimated age profiles of earnings ability from 65 to 67 and 69, respectively, are used as they are for the two representative agents. For Cases 68-75% and 70-75%, the estimated age profiles of earnings ability from 65 to 67 and 69, respectively, are weighted by 75% for both agents. For Cases 68-50% and 70-50%, the age profiles from 65 to 67 and 69, respectively, are weighted by 50% for both agents. In other words, Λ_s (the employment rate of age s) in Equation (3) are all unity before the starting age of pension benefits for Cases 68-100% and 70-100%. For Cases 68-75% and 70-75%, Λ_s are all unity until 64 and 0.75 for ages 65–67 and 65–69, respectively. For Cases 68-50% and 70-50%, Λ_s are all unity until 64 and 0.5 for ages 65–67 and 65–69, respectively.

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steady state, an average number of children to which a parent of the low-income class gives birth during the period from 18 to 40 is 0.0290 per year. For a parent of the high-income class, it is 0.0302 per year during the period from 22 to 40. We simply assume that a parent's available time is 16 h per day and that the childrearing time cost for one child is 1 h per day.

4.3.5. Taxes and expenditures

Tax rates on labor income, capital income, and inheritances are assumed to be fixed at the current levels (6.5%, 40%, and 10%, respectively) during the entire period until 2300. Tax rates on consumption are endogenously determined to satisfy Equations (24) and (35). General government expenditures, except for transfers to the public pension sector (πB_t) and government subsidies to childrearing (GS_t), are proportional to national income (Y_t), as indicated in Equation (27). The ratio of general expenditure to national income, g, is assigned 0.1 such that the endogenous tax rate on consumption is realistic and plausible in the 2022 initial steady state (i.e., 13.08%). The ratio is held constant at 0.1 throughout the entire period.

4.3.6. The public pension system

The public pension program is assumed to be a simple PAYG system similar to the current Japanese system. The benefit is assumed to comprise an earnings-related pension, although Japan's actual public pension system is two-tiered: a basic flat pension and an amount proportional to the average annual labor income for each household. General tax revenue finances half of the flat part, whereas contributions to the pension system fund both the remaining half and the entire proportional part. We assign the ratio (π) of the part financed by the tax transfer from the general account in Equation (26) as 0.25, taken from Oguro and Takahata (2013). The replacement ratio (θ) for public pension benefits in Equation (4) is equal to 40%, following Braun et al. (2009).

The age at which households start to receive public pension benefits (ST) is constant at 65 during the entire period for the baseline simulation. For six alternative simulation cases with a pension reform, it varies for each simulation case, as explained in the subsection 4.2. The compulsory retirement age (RE) is the starting age of public pension benefits (ST) minus 1. Thus, after households retire at the end of the year in which they reach compulsory retirement, they immediately start to receive pension benefits from the beginning of the next year.

4.3.7. Government deficits

Net government debt (D_t) is assumed to be proportional to national income to make our simulation feasible. The value of parameter d, which is the ratio of net public debt to national income as given in Equation (25), is assigned based on data from the Ministry of Finance (2023) and the Cabinet Office

(2023). After 2022, Japan's national income is expected to decrease as the population declines. Therefore, the assumption that net government debt is proportional to national income during the entire period implicitly implies that the government will successfully reduce future government deficits.

4.3.8. Intertemporal elasticity of substitution

Following İmrohoroğlu et al. (2017), the intertemporal elasticity of substitution (γ) in the individual utility function is set to 0.5. Our model also set the same value between the number of children and the consumption-leisure composite parameter, as in previous studies, such as Oguro, Takahata, and Shimasawa (2011) and Oguro and Takahata (2013).

4.3.9. Share parameter on consumption in utility

The value of the consumption share parameter, ϕ , in the utility function is assigned based on Nishiyama and Smetters (2005). Referring to Nishiyama and Smetters (2005), where $\phi = 0.47$, we set $\phi = 0.5$ in this paper. Consequently, in the 2022 initial steady state, an individual devotes, on average, 58.9% for the low-income class and 60.7% for the high-income class, of the available time endowment (of 16 h per day) to labor during their working years (ages 18–64 or 22–64 years).

4.3.10. Adjustment coefficient for discounting the future

The adjustment coefficient for discounting the future, δ , is set such that the capital–income ratio (K/Y) in the model, that is 2.46, approaches its plausible value, 2.5 which is estimated by Hansen and İmrohoroğlu (2016).

4.3.11. Technological progress

The technological progress of private production is significant because it greatly influences economic growth. Thus, careful attention should be paid to our assumptions. Technological progress is assumed to be 0 in the simulation, reflecting Japan's experience during the past two or three decades (see Ihori et al., 2006).

5. Simulation Results

We analyze alternative public pension reforms that raise the normal retirement age to 68 and 70 under three scenarios of the employment rate for elderly individuals in Japan. Based on the simulation results, we first address the effect of a rise in the normal retirement age on per-capita utility and the future population level. We then discuss the mechanism behind those findings.

5.1. Effect of a rise in the starting age for pension benefits on individual welfare

First, we evaluate the effect of a rise in the starting age for receiving pension benefits on individual welfare. Table 3 presents six alternative scenarios, which have two types of cases of the starting age (68 and 70) and three kinds of cases of elderly individual's employment rate (50%, 75%, and 100%). Table 7 shows leveled welfare gains for each individual based on the simulation results. The table shows that the 70-starting age cases have a larger welfare gain than the 68-starting age cases, indicating that a rise in the starting age for receiving pension benefits enhances the welfare gains. It also reveals that the higher the employment rate, the more the individual welfare improves. Consequently, Case 70-100% attains the highest individual's leveled welfare gain, equivalent to 25.377 million Japanese yen (JPY) (approximately193,000 U.S. dollars [USD] in 2022), a considerable amount for each individual.[5] In contrast, the welfare gain for Case 68-50% is only 10.265 million JPY (approximately 78,000 USD in 2022). These results suggest that the higher the starting age for receiving pensions and the higher the employment rate for elderly individuals, the more per-capita welfare improves. Therefore, we find that Case 70-100% is the most desirable from the viewpoint of per-capita welfare.

5.2. Effect of a rise in the starting age for pension benefits on future population

Next, we examine the effect of a rise in the starting age for receiving public pensions on the future demography. Figures 8 and 9 illustrate changes in the total population for three cases of different employment rates for elderly individuals (50%, 75%, and 100%) under the pension starting ages of 68 and 70, respectively, from the benchmark case with the 65-pension starting age. The higher the employment rate for elderly individuals, the larger the total population in the future. The qualitative

[5] The Cabinet Office (2023) estimated that the GDP of Japan in 2022 was 551.81 trillion yen. According to data from the Ministry of Internal Affairs and Communications (2023b), the number of the people aged 20–64 years was 58.66 million in 2022. We calculated the income per worker using these data and also derived the value for national GDP in 2022 in our model, yielding a conversion rate between actual amounts of yen and values in the model. Consequently, in 2022, unity in the model corresponded to 4.90035 million yen.

result is the same between the cases with the 68-starting age for public pensions and the 70-starting age, but the magnitude of the effect is greater for the case with the 70-starting age. Figure 8 illustrates that the total population in Case 68-100% continues to increase steadily, resulting in a 6.29% increase in 2200. The total population in Case 68-75% first gradually increases and peaks at a 1.50% increase around 2154; the increase then slightly shrinks over time. The total population in Case 68-50% first slightly increases but is almost flat; however, from 2110 onward, it starts to decrease, compared to the benchmark case with the 65-starting age. Figure 9 reveals that the total population in Case 70-100% increases by 9.47% in 2200. The total population in Case 70-75% peaks at a 2.05% increase around 2146; the increase slightly shrinks over time. The total population in Case 70-50% first slightly increases but is almost flat; however, from 2112 onward, it starts to decrease, compared to the benchmark case with the 65-starting age.

5.3. Findings and mechanism behind them

Based on the above simulation results, our findings are as follows. A rise in the starting age for receiving pension benefits enhances individual welfare, and the magnitude of the effect is greater in the case of the 70-pension starting age. Furthermore, a higher employment rate for elderly individuals improves individual welfare. The effect of a rise in the starting age on future demography depends significantly on the employment rate of elderly individuals. In the case of the 100%-employment rate, the future population will steadily increase; however, in the case of the 50%-employment rate, the current level in Japan, the future population starts to decrease after around 2110. Therefore, these results suggest that a rise in the starting age for pension benefits is desirable from the viewpoint of individual welfare, but from the viewpoint of the future population, it depends significantly on the employment rate of elderly individuals. If Japan’s employment rate remains at 50%, it will harm the total population in the long run. Therefore, when the starting age for pension benefits is raised, it is necessary to simultaneously increase the employment rate of elderly individuals as much as possible.

Figures 10 and 11 illustrate the percent changes in national income for three cases of different employment rates (50%, 75%, and 100%) for two scenarios of starting ages of 68 and 70, respectively, for pension benefits. Until 2200, the level of national income for the six simulation cases is higher throughout the entire period than that of the benchmark case with the 65-pension starting age. The higher the employment rate, the higher the level of national income. Case 68-100% attains a 15.4% increase in the national income in 2200; furthermore, Case 70-100% achieves a 24.3% increase in 2200. Conversely, the national income for Case 68-50% gradually increases and peaks at a 3.9% increase in 2083; the increase gradually shrinks over time. The national income for Case 70-50% gradually increases and peaks at a 6.2% increase in 2084; the increase gradually shrinks over time.

Figures 12 and 13 show the percent changes in capital stock and labor supply from the benchmark case under the scenario with the 70-pension starting age. The higher the employment rate of elderly individuals, the higher the level of capital stock. A rise in the starting age for receiving pension benefits means a reduction of public pension, which stimulates individual savings and increases capital stock. Case 70-100% achieves a 28.5% increase in 2200. Conversely, the capital stock for Case 70-50% gradually increases and peaks at 11.8% in 2101; the increase gradually shrinks over time. The higher the employment rate of elderly individuals, the higher the level of labor supply. This result suggests that a high employment rate means more labor supply, as measured by the efficiency units, because our model's employment rate represents the labor efficiency level. Case 70-100% attains a 21.9% increase in 2200. Conversely, the labor supply for Case 70-50% gradually increases and peaks at a 3.4% increase in 2082; the increase gradually shrinks over time.

Figures 14 and 15 illustrate the changes in interest rates and wage rates, respectively, from the benchmark case under the scenario with the 70-pension starting age (percentage-point changes for Figure 14; percent changes for Figure 15). Figure 14 reveals that the interest rate for Case 70-100% is first higher than that of the benchmark case but decreases after 2052. In contrast, Figure 15 denotes that the wage rate for Case 70-100% is first lower than that of the benchmark case but increases after 2052. These results suggest that the reform of Case 70-100% creates an additional considerable labor supply, which reduces the wage rate and increases the interest rate. As mentioned, Case 70-100% increases the capital stock over time and reduces the interest rate. After 2052, this effect of lowering interest rates through increases in capital stock might have exceeded the effect mentioned above of raising interest rates through increases in the labor supply. A possible reason for this is as follows. Initially, after the pension reform, many middle-aged and older generations suddenly faced a raised starting age for receiving pension benefits and a longer working period. Therefore, those generations might use the labor income from the additional working period mainly for consumption during their remaining lifetime. As time passes, more generations can gradually maximize their utility over the entire lifetime, including a longer working period, at younger ages. This situation may increase the portion of the additional labor income put into individual savings, which gradually promotes capital accumulation and significantly lowers interest rates.

For Case 70-50%, the additional labor supply measured by the efficiency units is not as large as in other cases. In the 70-50% case, the effect of increasing interest rates through increases in the labor supply is weak; thus, decreasing interest rates through increases in the capital stock is dominant. Consequently, the latter effect might have exceeded the former after 2030, earlier than in Case 70-100%. Additionally, Case 70-50% (with a low employment rate for elderly individuals) enhances the wage rate, thereby increasing opportunity costs for having children. As a result, the total population for Case 70-50% is increasingly smaller over time than in the other simulation cases, as illustrated in Figure 9.

Figure 16 denotes percentage-point changes in consumption tax rates from the benchmark case for Cases 70-50%, 70-75%, and 70-100%. The consumption tax rate for the three cases sharply drops until around 2080. From 2045 onward, the consumption tax rate for Case 70-100% is the lowest while the rate for Case 70-50% is the highest. Case 70-100% substantially increases the total population and national income, resulting in a higher tax revenue from labor income. This situation brings about lower consumption tax rates, which are endogenous variables in our model. This factor relatively improves individual welfare in Case 70-100%. Conversely, in Case 70-50%, the total population is smaller than the benchmark case after 2111 (Figure 9), resulting in a relatively low national income (Figure 11). This situation leads to a lower tax revenue from labor income, resulting in higher consumption tax rates. This factor relatively deteriorates the individual welfare in Case 70-50%.

Figure 17 illustrates percentage-point changes in contribution rates from the benchmark case for Cases 70-50%, 70-75%, and 70-100%. The contribution rate for the three cases sharply drops until around 2065. After 2060, the contribution rate for Case 70-100% is the lowest, while that for Case 70-50% is the highest. Figure 9 shows that Case 70-100% substantially increases the total population, which increases the young working population and reduces contribution rates under a PAYG social security system. This element also relatively improves individual welfare in Case 70-100%. Conversely, Figure 9 shows that the total population is relatively smaller in Case 70-50%. Consequently, reducing the young working population increases contribution rates under a PAYG social security system, which is also one factor that relatively deteriorates the individual welfare in Case 70-50%.

6. Conclusions

This paper evaluated the effects of a rise in the standard starting age of receiving pension benefits for a model parameterized to mimic certain features of the Japanese economy. We examined this situation from two viewpoints: individual welfare and future demography. Concretely, we examined the quantitative effects of a rise of the normal starting age from 65 to 68 and 70 on per-capita welfare and future population in an aging and depopulating Japan, using an extended lifecycle general equilibrium model with endogenous fertility. Our study focuses primarily on the difference in the employment rate for individuals aged 65 and above. The effects of two types of cases with the rising starting age (68 and 70) were quantitatively investigated under three scenarios with different employment rates (50%, 75%, and 100%) for elderly individuals during the transitional period, 2022–2300. We introduced an LSRA to calculate the per-capita welfare and evaluate the pure efficiency gains or losses of these pension reforms.

The two main findings of our analysis are as follows. First, a rise in the standard starting age for receiving pension benefits enhances individual welfare, and the magnitude of this effect is greater for the case with the 70-pension starting age than that with the 68-starting age. Furthermore, the higher the employment rate for individuals aged 65 and above, the more the individual welfare improves. Possible reasons for this result are as follows. A rise in the starting age for pension benefits means reducing public pensions and stimulates individual savings, which promotes capital formation, increases wage rates, and improves individual welfare. Additionally, a higher employment rate for elderly individuals means more labor supply, as measured by the efficiency units, which brings about a higher labor income and improves individual welfare.

Second, the effect of a rise in the normal starting age on future demography depends significantly on the employment rate of individuals aged 65 and above, especially in the long run. With the 100% employment rate for elderly individuals, the future population steadily increases. In the case of the current 50% employment rate, however, the future population will start to decrease after approximately 2110, compared to the benchmark case with the 65-pension starting age. A possible reason for this result is that a low employment rate for elderly individuals means a lower labor supply, as measured by the efficiency units, which increases the wage rates and enhances opportunity costs for having children.

Finally, we discuss policy implications based on the simulation results. The above main findings suggest that a rise in the standard starting age for pension benefits is desirable from the viewpoint of individual welfare; however, from the viewpoint of future population, it depends significantly on the employment of individuals aged 65 and above. A high employment rate for elderly individuals improves welfare and the future population; however, when Japan's employment rate remains at 50%, it would harm the total population level in the long run, compared to the benchmark case with the 65-starting age for pension benefits. Therefore, if the starting age for pension benefits increases, it would be desirable to simultaneously increase the employment rate of elderly individuals, for example, up to 75%.

Appendix A: Model for the High-Income Class (University Graduates)

Here, we describe the household behavior of the high-income class household (i.e., university graduates).

A.1 Household behavior

Each agent enters the economy as a decision-making unit and starts to work at age 22 years, and lives to a maximum age of 105 years with uncertainty of death. The children aged 0–17 or 0–21 only consume, involving childrearing costs for their parent. The probability of a household born in year t, surviving until s, can be expressed by

$$p_s^{t(U)} = \prod_{j=22}^{s-1} q_{j+1|j}^t. \qquad (1)'$$

Each agent who begins its economic life at age 22 chooses perfect-foresight consumption paths ($C_s^{t(U)}$), leisure paths ($l_s^{t(U)}$), and the number of born children ($n_s^{t(U)}$) to maximize a time-separable utility function of the form:

$$U^{t(U)} = \frac{1}{1-\frac{1}{\gamma}}\left[\alpha^{(U)} \sum_{s=22}^{40} p_s^{t(U)} (1+\delta)^{-(s-22)} \left(n_s^{t(U)}\right)^{1-\frac{1}{\gamma}} + (1-\alpha^{(U)}) \sum_{s=22}^{105} p_s^{t(U)} (1+\delta)^{-(s-22)} \left\{\left(C_s^{t(U)}\right)^{\varphi} \left(l_s^{t(U)}\right)^{1-\varphi}\right\}^{1-\frac{1}{\gamma}}\right]. \qquad (2)'$$

where $C_s^{t(U)}$, $l_s^{t(U)}$ and $n_s^{t(U)}$ are respectively consumption, leisure and the number of children to bear (only in the first 19 periods of the life) for an agent born in year t, of age s. $\alpha^{(U)}$ is the utility weight of the number of children relative to the consumption–leisure composite.

Letting $A_s^{t(U)}$ be capital holdings for the agent born in year t, of age s, maximization of Equation (2)' is subject to a lifetime budget constraint defined by the sequence:

$$A_{s+1}^{t(U)} = \{1 + r_{t+s}(1-\tau^r)\}A_s^{t(U)} + (1 - \tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(U)} \{1 - l_s^{t(U)} - tc_s^{t(U)}(n_s^{t(U)})\}\Lambda_s + a_s^{t(U)} - or_s^{t(U)} + b_s^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) - (1+\tau_t^c)C_s^{t(U)} - (1-m)(1+\tau_t^c)\Phi_s^{t(U)} - m(1+\tau_t^c)\Phi_s^{t(H)} \quad .$$

(3)'

There are no liquidity constraints, and thus the assets can be negative. An individual's earnings ability

$e_s^{(U)}$ is an exogenous function of age, and Λ_s denotes the employment rate of age s.

The pension benefit is assumed to comprise only an earnings-related pension:

$$b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)=\begin{cases}\theta H^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) & (s\geq ST)\\ 0 & (s<ST)\end{cases}, \quad (4)'$$

where

$$H^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)=\frac{1}{RE-21}\sum_{s=22}^{RE}w_{t+s}e_s^{(U)}\{1-l_s^{t(U)}-tc_s^t(n_u^{t(U)})\}. \quad (5)'$$

The average annual labor income for each agent is $H^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)$, and the weight coefficient of the part proportional to $H^{t(U)}$ is θ. The symbol $b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)$ in Equation (3)' signifies that the amount of public pension benefit is a function of the age profile of labor supply, $\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}$.

A parent is assumed to bear children and expend for them until they become independent of their parent, namely, during the period when they are from zero to 21 years old. Here, note that the children aged below 22 years old do not conduct an economic activity independently, and only childrearing cost for their parent arises until they become independent of their parent. The financial costs for rearing the children when the parent born in year t is s years old are represented by $\Phi_s^{t(U)}$ and $\Phi_s^{t(H)}$, which are the cost for the children who will become university graduates and high school graduates, respectively:

$$\Phi_s^{t(U)}=\begin{cases}\sum_{k=22}^{s}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=22,23,\cdots,40)\\ \sum_{k=22}^{40}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=41,42,43)\\ \sum_{k=s-21}^{40}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=44,45,\cdots,61)\end{cases}, \quad (6)'$$

$$\Phi_s^{t(U)}=0\ \ (s=62,63,\cdots,105), \quad (7)'$$

$$\Phi_s^{t(H)}=\begin{cases}\sum_{k=22}^{s}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=22,23,\cdots,39)\\ \sum_{k=s-17}^{40}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=40,41,\cdots,57)\end{cases}, \quad (8)'$$

$$\Phi_s^{t(H)}=0\ \ (s=58,59,\cdots,105), \quad (9)'$$

$$\xi^{t(U)}=\beta NW^{t(U)}. \quad (10)'$$

The time cost for rearing the children when the parent born in year t is s years old is represented by

$$tc_s^t=\mu n_s^{t(U)}. \quad (11)'$$

When $BQ_t^{(U)}$ is the sum of bequests inherited by the high-income class households at time t, the bequest to be inherited by each high-income class household is defined by

$$a_s^{t(U)}=\frac{(1-\tau^h)BQ_{t+s}^{(U)}}{E_{t+s}^{(U)}}, \quad (12)'$$

where $E_t^{(U)}$ is the number of the high-income class households conducting an economic activity independently, aged 22 and above, and

$$BQ_t^{(U)} = \sum_{s=22}^{105}(N_s^{t-s-1(U))} - N_{s+1}^{t-s-1(U)})A_{s+1}^{t-s-1(U)}. \tag{13)'}$$

The number of the generation born in year t, of age s, is represented by

$$N_s^{t(U)} = p_s^{t(U)} N_0^{t(U)}. \tag{14)'}$$

When $OR_t^{(U)}$ is the sum of childrearing costs incurred by the high income class households at time t, the childrearing cost for orphans for each high income class household is defined by

$$or_s^{t(U)} = \frac{OR_{t+s}^{(U)}}{E_{t+s}^{(U)}}, \tag{15)'}$$

where

$$OR_t^{(U)} = (1-m)\sum_{s=22}^{61}(N_{s-1}^{t-s(U)} - N_s^{t-s(U)})\Phi_s^{t-s(U)} + m\sum_{s=22}^{57}(N_{s-1}^{t-s(U)} - N_s^{t-s(U)})\Phi_s^{t-s(H)}. \tag{16)'}$$

When we consider the utility maximization problem over time for each agent, besides the flow budget constraint represented by Equation (3)', the following constraint is imposed:

$$\begin{cases} 0 \le l_s^{t(U)} \le 1 - tc_s^t(n_s^{t(U)})(22 \le s \le RE) \\ \quad l_s^{t(U)} = 1\ (RE+1 \le s \le 105) \end{cases}. \tag{17)'}$$

Each individual maximizes Equation (2)' subject to Equations (3)' and (17)' (see Appendix C for further details). From the utility maximization problem, the equation expressing the evolution of the number of children over time for each individual is characterized by

$$W_s^{t(U)} = \left(\frac{p_{s-1}^{t(U)}}{p_s^{t(U)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right] W_{s-1}^{t(U)}, \tag{18)'}$$

$$W_s^{t(U)} = \frac{\alpha^{(U)} k^{1-\frac{1}{\gamma}}(n_s^{t(U)})^{-\frac{1}{\gamma}}}{(1+\tau_{t+s}^c)[(1-m)\sum_{g=0}^{21}\Omega_{s,g}^t \xi^{t(U)}(1-\rho)+m\sum_{g=0}^{17}\Omega_{s,g}^t \xi^{t(U)}(1-\rho)]}, \tag{19)'}$$

where $\Omega_{s,0}^t$=1 for $g=0$, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g}\{1+r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$.

Similarly, that for the consumption–leisure composite is represented by

$$V_s^{t(U)} = \left(\frac{p_{s-1}^{t(U)}}{p_s^{t(U)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right] V_{s-1}^{t(U)}, \tag{20)'}$$

$$V_s^{t(U)} = \frac{(1-\alpha^{(U)})\left\{(C_s^{t(U)})^{\varphi}(l_s^{t(U)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}}\varphi(C_s^{t(U)})^{\varphi-1}(l_s^{t(U)})^{1-\varphi}}{1+\tau_t^c}. \tag{21)'}$$

Appendix B: The Utility Maximization Problem for the Low-Income Class

The utility maximization problem over time for each low-income class household in Section 2 is regarded as the maximization of $U^{t(H)}$in Equation (2) subject to Equations (3) and (17). Let the Lagrange function be

$$L^{t(H)} = U^{t(H)} + \sum_{s=18}^{105} \lambda_s^{t(H)} \Big[-A_{s+1}^{t(H)} + \{1 + r_{t+s}(1-\tau^r)\}A_s^{t(H)} + [1 - \tau^w - \tau_{t+s}^p]w_{t+s}e_s^{(H)}\{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\}\Lambda_s + a_s^{t(H)} - or_s^{t(H)} + b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) - (1 + \tau_{t+s}^c)C_s^{t(H)} - (1-m)(1+\tau_{t+s}^c)\Phi_s^{t(H)} - m(1+\tau_{t+s}^c)\Phi_s^{t(U)}\Big] + \sum_{s=18}^{RE} \eta_s^{t(H)}\{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\}$$

(B.1)

where $\lambda_s^{t(H)}$ and $\eta_s^{t(H)}$ represent the Lagrange multiplier for Equations (3) and (17), respectively.

The first-order conditions on the number of children $n_s^{t(H)}$, consumption $C_s^{t(H)}$, leisure $l_s^{t(H)}$, and assets $A_{s+1}^{t(H)}$ for s =18, 19, …, 105 can be expressed by

$$p_s^{t(H)}\alpha^{(H)}(1+\delta)^{-(s-18)}(n_s^{t(H)})^{-\frac{1}{\gamma}} = \lambda_s^{t(H)}\Big\{\mu[1 - \tau^w - \tau_{t+s}^p]w_{t+s}e_s^{(H)}\Lambda_s + (1-m)(1+\tau_t^c)\sum_{g=0}^{17}\Omega_{s,g}^t\xi^{t(H)}(1-\rho) + m(1+\tau_t^c)\sum_{g=0}^{21}\Omega_{s,g}^t\xi^{t(H)}(1-\rho)\Big\} + \mu\sum_{k=ST}^{105}\lambda_k^{t(H)}\frac{\theta w_{t+s}e_s^{(H)}\Lambda_s}{RE-19} + \mu\eta_s^{t(H)}, \quad \text{(B.2)}$$

where $\Omega_{s,0}^t$=1 for g =0, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g}\{1 + r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$,

$$p_s^{t(H)}(1-\alpha^{(H)})(1+\delta)^{-(s-18)}\left\{(C_s^{t(H)})^{\varphi}(l_s^{t(H)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}}\varphi(C_s^{t(H)})^{\varphi-1}(l_s^{t(H)})^{1-\varphi} = \lambda_s^{t(H)}(1+\tau_{t+s}^c), \quad \text{(B.3)}$$

$$p_s^{t(H)}(1-\alpha^{(H)})(1+\delta)^{-(s-18)}\left\{(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{1-\phi}\right\}^{-\frac{1}{\gamma}}(1-\phi)(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{-\phi} = \lambda_s^{t(H)}\left\{(1 - \tau^w - \tau_{t+s}^p)w_{t+s}e_s^{(H)}\Lambda_s\right\} + \sum_{k=ST}^{105}\lambda_k^{t(H)}\frac{\theta w_{t+s}e_s^{(H)}\Lambda_s}{RE-19} + \eta_s^{t(H)} \quad (s \le RE), \quad \text{(B.4)}$$

$$\lambda_s^{t(H)} = \{1 + r_{t+s}(1-\tau^r)\}\lambda_{s+1}^{t(H)}, \quad \text{(B.5)}$$

$$\eta_s^{t(H)}\{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\} = 0 \quad (s \le RE), \quad \text{(B.6)}$$

$$1 - l_s^{t(H)} = 0 \quad (s > RE), \quad \text{(B.7)}$$

$$\eta_s^{t(H)} \ge 0. \quad \text{(B.8)}$$

The combination of Equations (B.2) and (B.5) produces Equations (18) and (19). If the initial value, $n_{18}^{t(H)}$, is given, the initial value, $W_{18}^{t(H)}$, can be derived from Equation (19). If the value, $W_{18}^{t(H)}$, is specified, the value of each age, $W_s^{t(H)}$, can be derived from Equation (18), which generates the value of each age, $n_s^{t(H)}$. If the value, $n_s^{t(H)}$, is specified, the child rearing cost for lifetime is calculated, which gives the lifetime budget constraint represented by Equation (B.10).

The combination of Equations (B.3) and (B.5) produces Equations (20) and (21). If the initial value,

$V_{18}^{t(H)}$, is specified, the value of each age, $V_s^{t(H)}$, can be derived from Equation (20). If $V_s^{t(H)}$ is specified, the values of consumption, $C_s^{t(H)}$, and leisure, $l_s^{t(H)}$, at each age are obtained in the method that follows.

For s =18, 19, …, *RE*, the combination of Equations (B.3) and (B.4) yields the following expression:

$$C_s^{t(H)} = \left[\frac{\varphi\left\{(1-\tau^w-\tau_{t+s}^p)w_{t+s}e_s^{(H)}\Lambda_s+\sum_{k=ST}^{105}\frac{\lambda_k^{t(H)}}{\lambda_s^{t(H)}}\frac{\theta w_{t+s}e_s^{(H)}\Lambda_s}{RE-20}+\frac{\eta_s^{t(H)}}{\lambda_s^{t(H)}}\right\}}{(1-\varphi)(1+\tau_{t+s}^c)}\right] l_s^{t(H)}. \tag{B.9}$$

If the value of $l_s^{t(H)}$ is given under $\eta_s^{t(H)}$=0, the value of $C_s^{t(H)}$ can be obtained using a numerical method, and then the value of $V_s^{t(H)}$ can be derived from Equation (21). The value of $l_s^{t(H)}$ is chosen so that the value of $V_s^{t(H)}$ obtained in the simulation is the closest to that calculated by evolution from $V_{18}^{t(H)}$ through Equation (20). If the value of $l_s^{t(H)}$ chosen is unity or higher, the value of $C_s^{t(H)}$ is obtained from Equation (21) under $l_s^{t(H)}$=1. If it is less than unity, the value of $C_s^{t(H)}$ is derived from Equation (B.9).

For s =*RE*+1, *RE*+2, …, 105, the condition of $l_s^{t(H)}$=1 leads to the following equation:

$$V_s^{t(H)} = \frac{(1-\alpha^{(H)})\varphi(C_s^{t(H)})^{-\frac{\varphi}{\gamma}+\varphi-1}}{1+\tau_{t+s}^c}. \tag{21}”$$

The value of $C_s^{t(H)}$ is chosen to satisfy this equation.

From Equation (3) and the terminal condition $A_{18}^{t(H)}$=$A_{106}^{t(H)}$=0, the lifetime budget constraint for an individual (= $NW^{t(H)}$) is derived:

$$\sum_{s=18}^{RE}\Psi_s^{t(H)}[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(H)}\{1-l_s^{t(H)}-tc_s^t(n_s^{t(H)})\}\Lambda_s+\sum_{s=ST}^{105}\Psi_s^{t(H)}b_s^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)+\sum_{s=18}^{105}\Psi_s^{t(H)}(a_s^{t(H)}-or_s^{t(H)})=\sum_{s=18}^{105}\Psi_s^{t(H)}(1+\tau_{t+s}^c)C_s^{t(H)}+(1-m)\sum_{s=18}^{35}\sum_{k=18}^{s}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+(1-m)\sum_{s=36}^{40}\sum_{k=s-17}^{s}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+(1-m)\sum_{s=41}^{57}\sum_{k=s-17}^{40}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+m\sum_{s=18}^{39}\sum_{k=18}^{s}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+m\sum_{s=40}^{61}\sum_{k=s-21}^{40}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}, \tag{B.10}$$

where $\Psi_{18}^{t(H)}$=1 for s =18, $\Psi_s^{t(H)} = (\prod_{u=19}^{s}\{1+r_{t+u}(1-\tau^r)\})^{-1}$ for s =19, 20, …, 105.

Appendix C: The Utility Maximization Problem for the High-Income Class

The utility maximization problem over time for each high-income class household in Appendix A is regarded as the maximization of $U^{t(U)}$ in Equation (2)' subject to Equations (3)' and (17)'. Let the Lagrange function be

$$L^{t(U)} = U^{t(U)} + \sum_{s=22}^{105} \lambda_s^{t(U)} \Big[-A_{s+1}^{t(U)} + \{1 + r_{t+s}(1-\tau^r)\} A_s^{t(U)} + [1 - \tau^w - \tau_{t+s}^p] w_{t+s} e_s^{(U)} \{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\} \Lambda_s + a_s^{t(U)} - or_s^{t(U)} + b_s^{t(U)} \left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE} \right) - (1 + \tau_{t+s}^c) C_s^{t(U)} - (1-m)(1+\tau_t^c) \Phi_s^{t(U)} - m(1+\tau_t^c) \Phi_s^{t(H)} \Big] + \sum_{s=22}^{RE} \eta_s^{t(U)} \{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\}, \quad \text{(C.1)}$$

where $\lambda_s^{t(U)}$ and $\eta_s^{t(U)}$ represent the Lagrange multiplier for Equations (3)' and (17)', respectively.

The first-order conditions on the number of children $n_s^{t(U)}$, consumption $C_s^{t(U)}$, leisure $l_s^{t(U)}$, and assets $A_{s+1}^{t(U)}$ for s =22, 23, …, 105 can be expressed by

$$p_s^{t(U)} \alpha^{(U)} (1+\delta)^{-(s-22)} (n_s^{t(U)})^{-\frac{1}{\gamma}} = \lambda_s^{t(U)} \left\{ \mu [1 - \tau^w - \tau_{t+s}^p] w_{t+s} e_s^{(U)} \Lambda_s + (1-m)(1+\tau_t^c) \sum_{g=0}^{21} \Omega_{s,g}^t \xi^{t(U)} (1-\rho) + m(1+\tau_t^c) \sum_{g=0}^{17} \Omega_{s,g}^t \xi^{t(U)} (1-\rho) \right\} + \mu \sum_{k=ST}^{105} \lambda_k^{t(U)} \frac{\theta w_{t+s} e_s^{(U)} \Lambda_s}{RE-21} + \mu \eta_s^{t(U)}, \quad \text{(C.2)}$$

where $\Omega_{s,0}^t$=1 for g =0, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g} \{1 + r_{t+s-1+k}(1-\tau^r)\} \right)^{-1}$,

$$p_s^{t(U)} (1-\alpha^{(U)}) (1+\delta)^{-(s-22)} \left\{ (C_s^{t(U)})^{\varphi} (l_s^{t(U)})^{1-\varphi} \right\}^{-\frac{1}{\gamma}} \varphi (C_s^{t(U)})^{\varphi-1} (l_s^{t(U)})^{1-\varphi} = \lambda_s^t (1+\tau_{t+s}^c), \quad \text{(C.3)}$$

$$p_s^{t(U)} (1-\alpha^{(U)}) (1+\delta)^{-(s-22)} \left\{ (C_s^{t(U)})^{\phi} (l_s^{t(U)})^{1-\phi} \right\}^{-\frac{1}{\gamma}} (1-\phi) (C_s^{t(U)})^{\phi} (l_s^{t(U)})^{-\phi} = \lambda_s^{t(U)} \left\{ (1 - \tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(U)} \Lambda_s \right\} + \sum_{k=ST}^{105} \lambda_k^{t(U)} \frac{\theta w_{t+s} e_s^{(U)} \Lambda_s}{RE-21} + \eta_s^{t(U)} \quad (s \le RE), \quad \text{(C.4)}$$

$$\lambda_s^{t(U)} = \{1 + r_{t+s}(1-\tau^r)\} \lambda_{s+1}^{t(U)}, \quad \text{(C.5)}$$

$$\eta_s^{t(U)} \{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\} = 0 \quad (s \le RE), \quad \text{(C.6)}$$

$$1 - l_s^{t(U)} = 0 \quad (s > RE), \quad \text{(C.7)}$$

$$\eta_s^{t(U)} \ge 0. \quad \text{(C.8)}$$

The combination of Equations (C.2) and (C.5) produces Equations (18)' and (19)'. If the initial value, $n_{22}^{t(U)}$, is given, the initial value, $W_{22}^{t(U)}$, can be derived from Equation (19)'. If the value, $W_{22}^{t(U)}$, is specified, the value of each age, $W_s^{t(U)}$, can be derived from Equation (18)', which generates the value of each age, $n_s^{t(U)}$. If the value, $n_s^{t(U)}$, is specified, the child rearing cost for lifetime is calculated, which gives the lifetime budget constraint represented by Equation (C.10).

The combination of Equations (C.3) and (C.5) produces Equations (20)’ and (21)’. If the initial value, $V_{22}^{t(U)}$, is specified, the value of each age, $V_s^{t(U)}$, can be derived from equation (20)’. If $V_s^{t(U)}$ is specified, the values of consumption, $C_s^{t(U)}$, and leisure, $l_s^{t(U)}$, at each age are obtained in the method that follows.

For s =22, 23, …, *RE*, the combination of Equations (C.3) and (C.4) yields the following expression:

$$C_s^{t(U)} = \left[\frac{\varphi \left\{ (1-\tau^w-\tau_{t+s}^p) w_{t+s} e_s^{(U)} \Lambda_s + \sum_{k=ST}^{105} \frac{\lambda_k^{t(U)}}{\lambda_s^{t(U)}} \frac{\theta w_{t+s} e_s^{(U)} \Lambda_s}{RE-21} + \frac{\eta_s^{t(U)}}{\lambda_s^{t(U)}} \right\}}{(1-\varphi)(1+\tau_{t+s}^c)} \right] l_s^{t(U)}. \quad \text{(C.9)}$$

If the value of $l_s^{t(U)}$ is given under $\eta_s^t = 0$, the value of $C_s^{t(U)}$ can be obtained using a numerical method, and then the value of $V_s^{t(U)}$ can be derived from Equation (21)’. The value of $l_s^{t(U)}$ is chosen so that the value of $V_s^{t(U)}$ obtained in the simulation is the closest to that calculated by evolution from $V_{22}^{t(U)}$ through Equation (20)’. If the value of $l_s^{t(U)}$ chosen is unity or higher, the value of $C_s^{t(U)}$ is obtained from Equation (21)’ under $l_s^{t(U)}$=1. If it is less than unity, the value of $C_s^{t(U)}$ is derived from Equation (C.9).

For s =*RE*+1, *RE*+2, …, 105, the condition of $l_s^{t(U)}$=1 leads to the following equation:

$$V_s^{t(U)} = \frac{(1-\alpha^{(U)})\varphi (C_s^{t(U)})^{-\frac{\varphi}{\gamma}+\varphi-1}}{1+\tau_{t+s}^c}. \quad \text{(21)'''}$$

The value of $C_s^{t(U)}$ is chosen to satisfy this equation.

From Equation (3)’ and the terminal condition $A_{22}^{t(U)}$=$A_{106}^{t(U)}$=0, the lifetime budget constraint for an individual (=$NW^{t(U)}$) is derived:

$$\sum_{s=22}^{RE} \Psi_s^{t(U)}[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(U)}\{1-l_s^{t(U)}-tc_s^t(n_s^{t(U)})\}\Lambda_s$$

$$+\sum_{s=ST}^{105} \Psi_s^{t(U)} b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)+\sum_{s=22}^{105} \Psi_s^{t(U)}(a_s^{t(U)}-or_s^{t(U)})$$

$$=\sum_{s=22}^{105} \Psi_s^{t(U)}(1+\tau_{t+s}^c)C_s^{t(U)}+(1$$

$$-m)\sum_{s=22}^{40}\sum_{k=22}^{s} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}+(1$$

$$-m)\sum_{s=41}^{43}\sum_{k=22}^{40} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}$$

$+(1-m)\sum_{s=44}^{61}\sum_{k=s-21}^{40} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}+m\sum_{s=22}^{39}\sum_{k=22}^{s} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-$ $\rho)n_k^{t(U)}+m\sum_{s=40}^{57}\sum_{k=s-17}^{40} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}$, (C.10)

where $\Psi_{22}^{t(U)}$=1 for s =22, $\Psi_s^{t(U)}=(\prod_{u=23}^{s}\{1+r_{t+u}(1-\tau^r)\})^{-1}$ for s =23, 24, …, 105.

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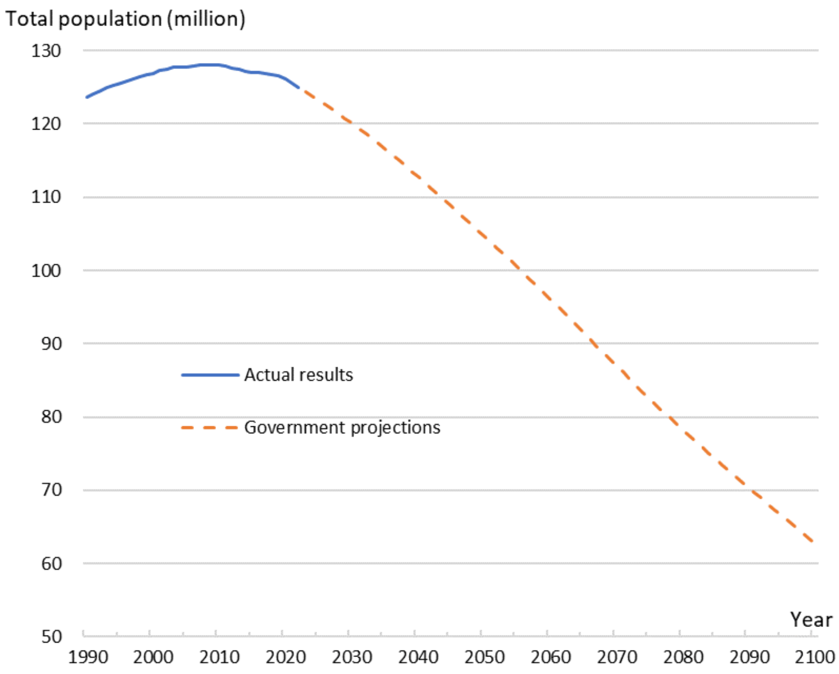

Figure 1 Total population in Japan: Actual results and projections

Source: Statistics Bureau of Japan (2023a) for actual results until 2022 on the total population; National Institute of Population and Social Security Research (2023) for government projections after 2022

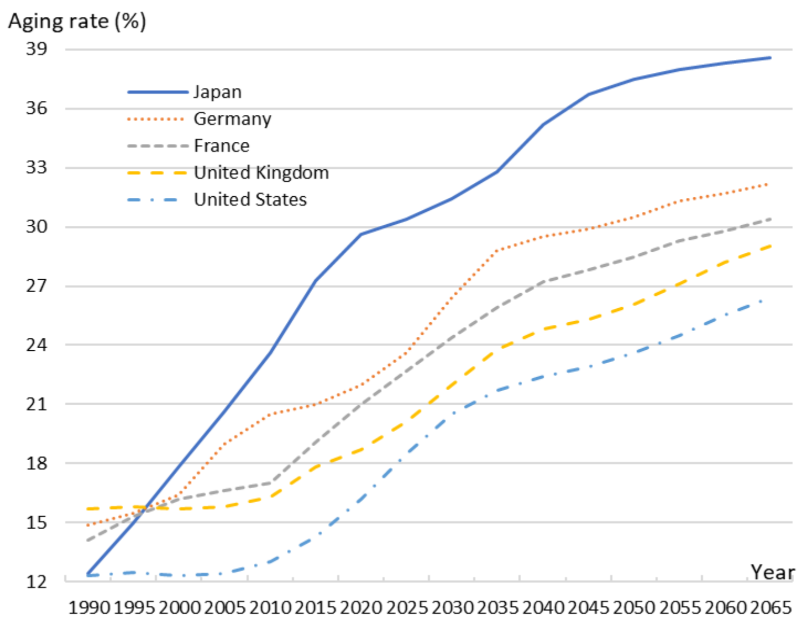

Figure 2 Aging rate transition for five advanced countries

Source: United Nations (2023)

Note: Aging rates indicate the ratios of the people aged 65 and above to the total population.

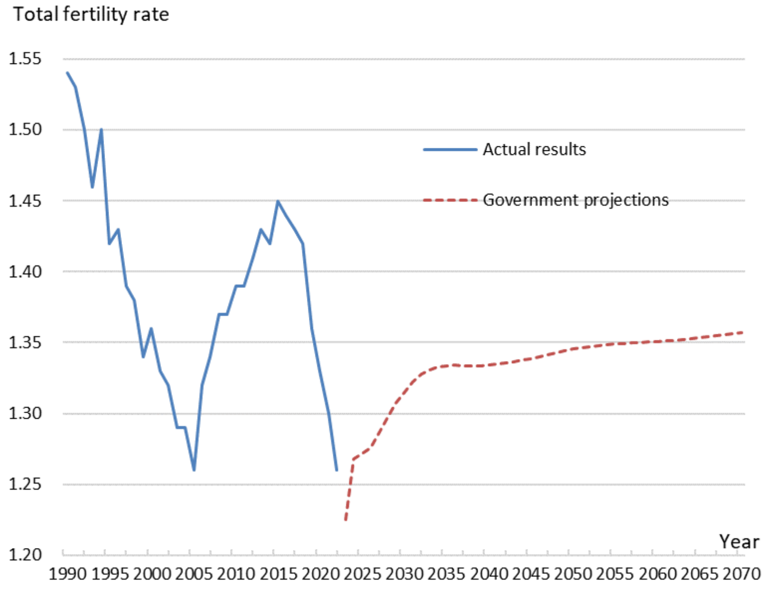

Figure 3 Total fertility rate in Japan: Actual results and projections

Source: Ministry of Health, Labour and Welfare (1991–2023a) for actual results until 2022 on the total fertility rate; National Institute of Population and Social Security Research (2023) for government projections after 2022

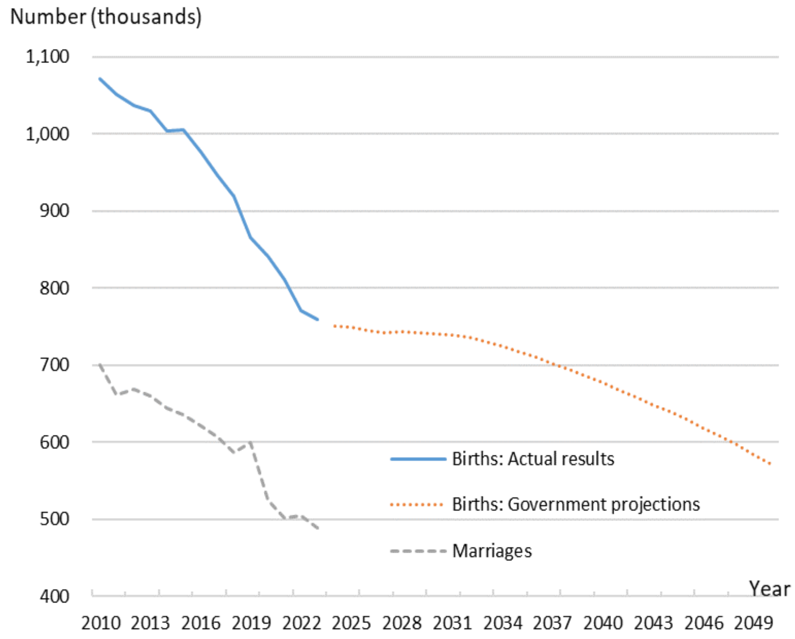

Figure 4 Number of Births and Marriages in Japan

Source: Ministry of Health, Labour and Welfare (2011–2024) for actual results until 2023 on births and marriages; National Institute of Population and Social Security Research (2023) for government projections on births after 2023

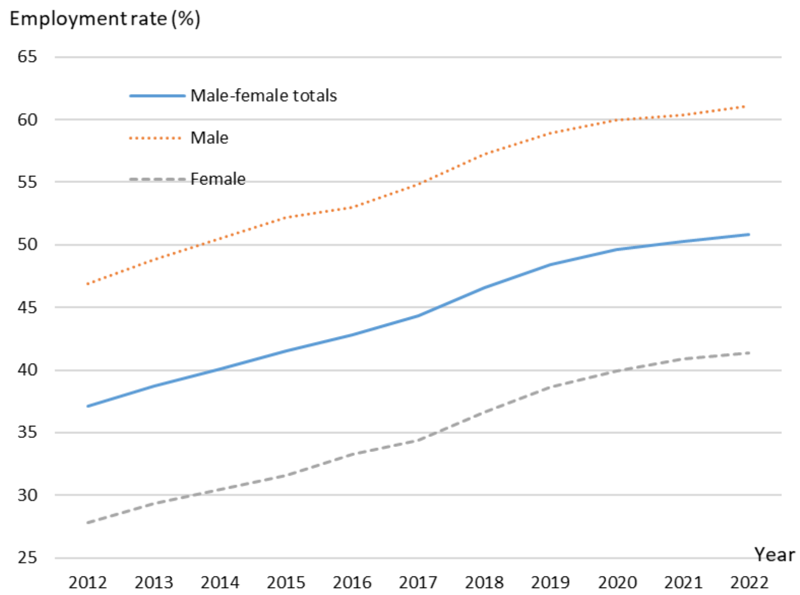

Figure 5 Employment rate for the elderly aged 65–69

Source: Ministry of Internal Affairs and Communications (2023)

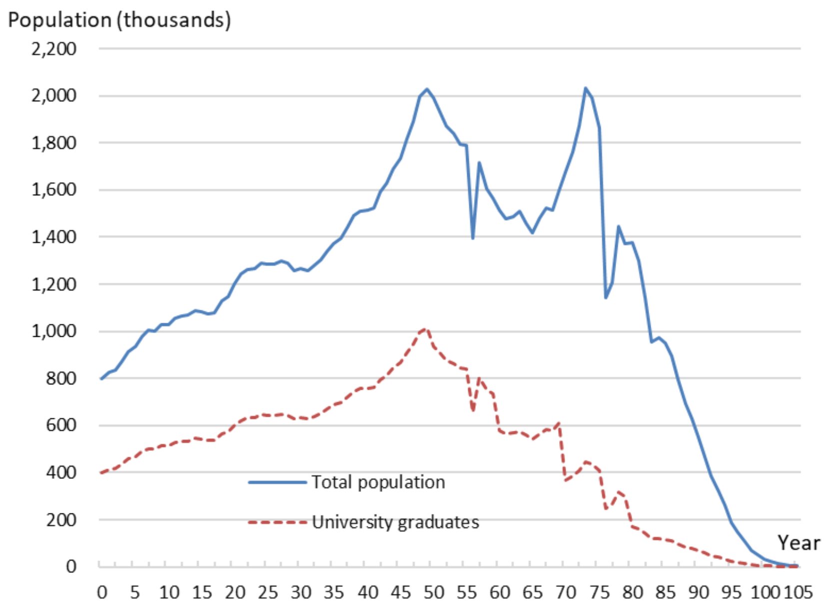

Figure 6 Age–population distribution in the 2022 initial steady state

Notes: The vertical gap between the total population and the number of university graduates is the number of high school graduates for each age. For young people unsure if they will be (just) high school graduates or university graduates, we assume 50/50.

Source: Statistics Bureau of Japan (2023a)

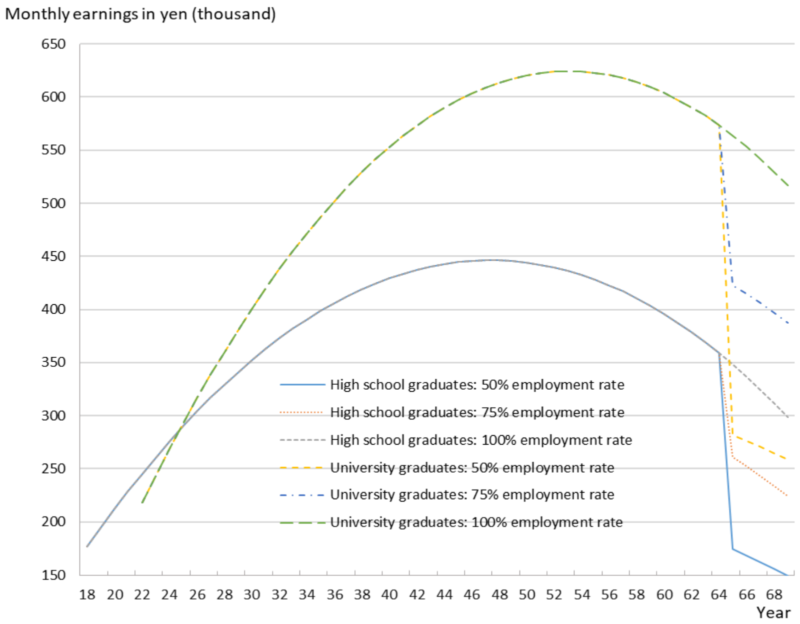

Figure 7 Age earnings profiles for two representative agents with different employment rates (50%, 75%, and 100%)

Note: Age earnings profiles consider different employment rates after age 64 for each simulation case.

Source: The profiles are estimated from the Ministry of Health, Labour and Welfare (2014–2023b) for the 2013–2022 period.

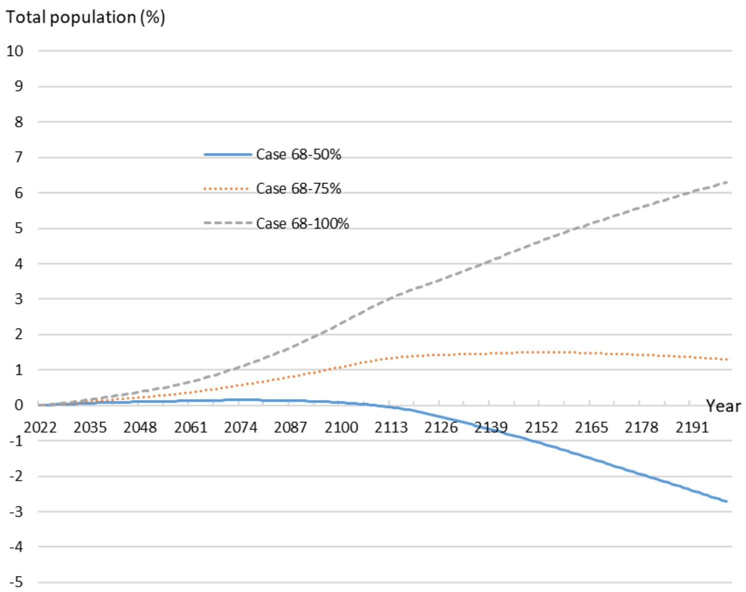

Figure 8 Changes in total population for three cases of different employment rates (50%, 75%, and 100%) under the 68-starting age for pension benefits

Note: The figure shows changes in total population from the benchmark case with the 65-starting age for pension benefits.

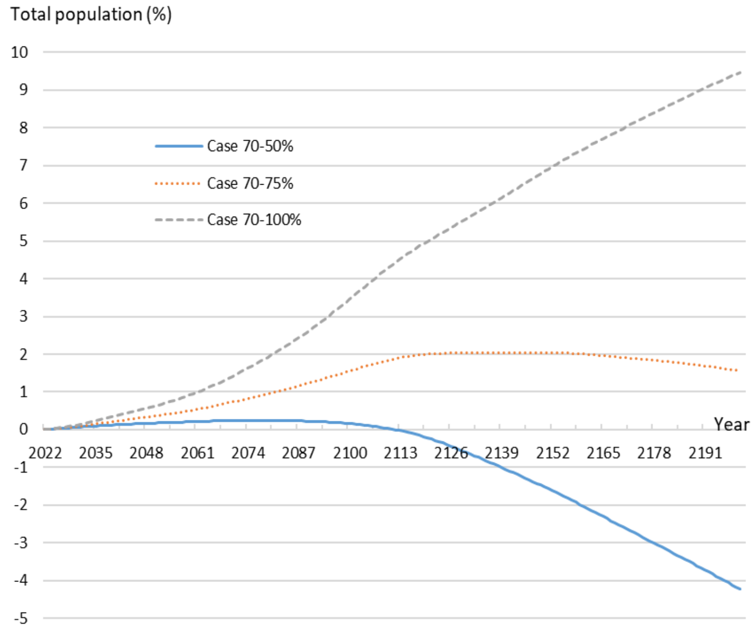

Figure 9 Changes in total population for three cases of different employment rates (50%, 75%, and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in total population from the benchmark case with the 65-starting age for pension benefits.

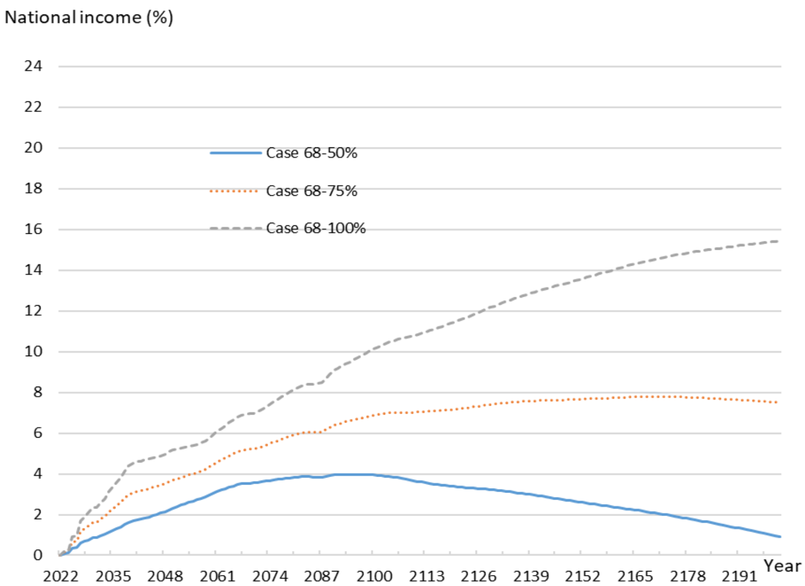

Figure 10 Changes in natinal income for three cases of different employment rates (50%, 75%, and 100%) under the 68-starting age for pension benefits

Notes: The figure shows changes in national income from the benchmark case with the 65-starting age for pension benefits.

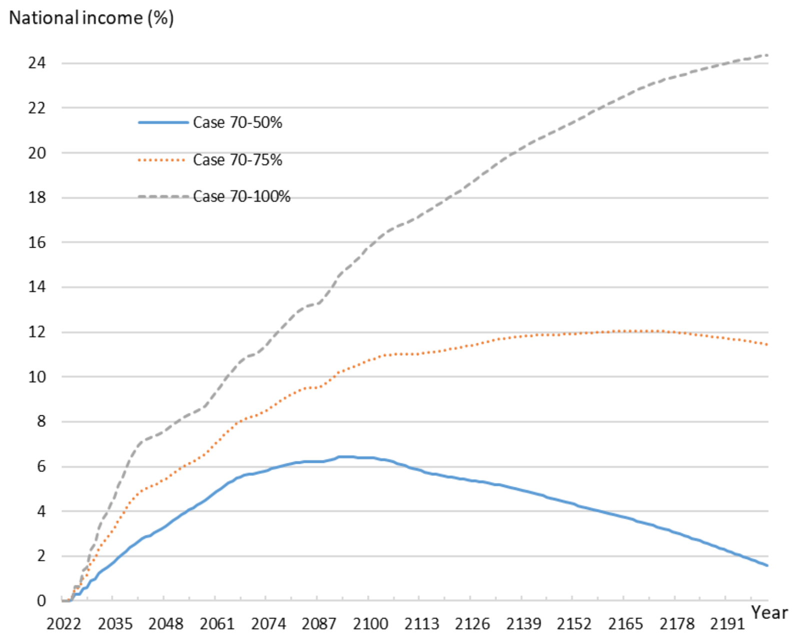

Figure 11 Changes in national income for three cases of different employment rates (50%, 75%, and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in national income from the benchmark case with the 65-starting age for pension benefits.

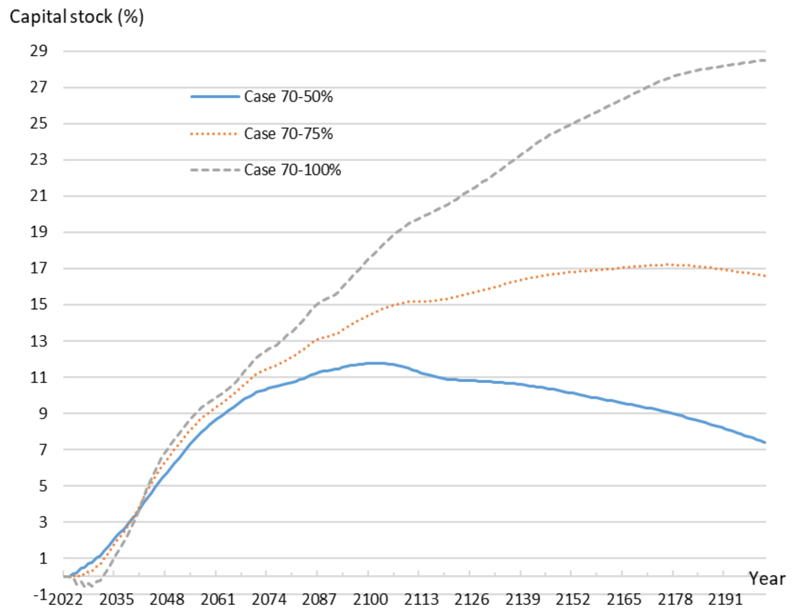

Figure 12 Changes in capital stock for three cases of different employment rates (50%, 75%, and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in capital stock from the benchmark case with the 65-starting age for pension benefits.

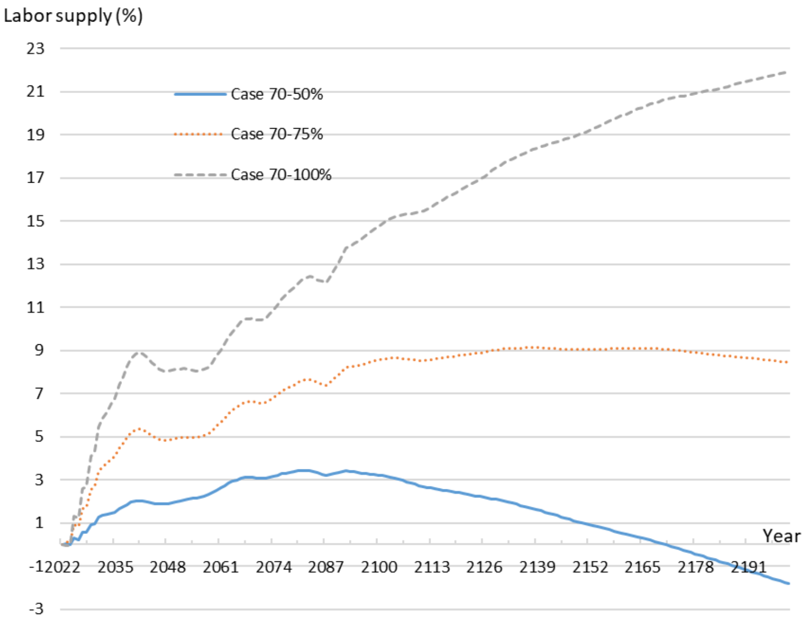

Figure 13 Changes in labor supply for three cases of different employment rates (50%, 75%, and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in labor supply from the benchmark case with the 65-starting age for pension benefits.

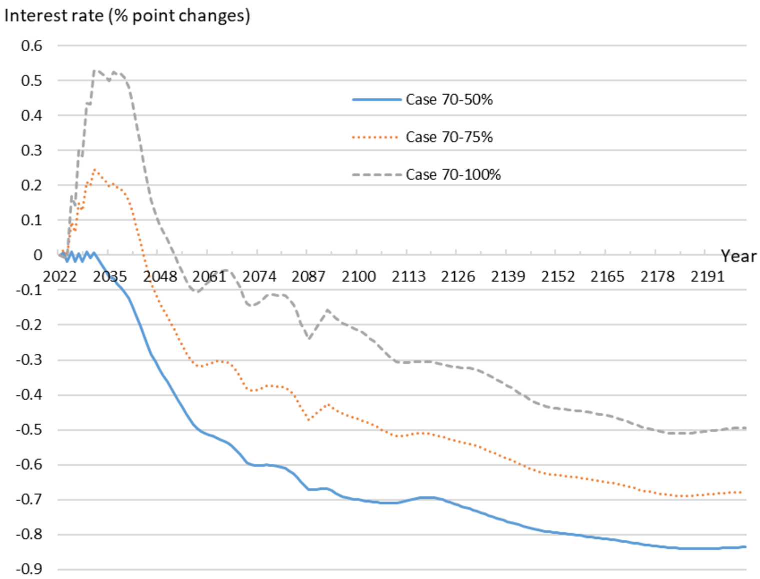

Figure 14 Changes in interest rates for three cases of different employment rates (50%, 75%, and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in interest rates from the benchmark case with the 65-starting age for pension benefits.

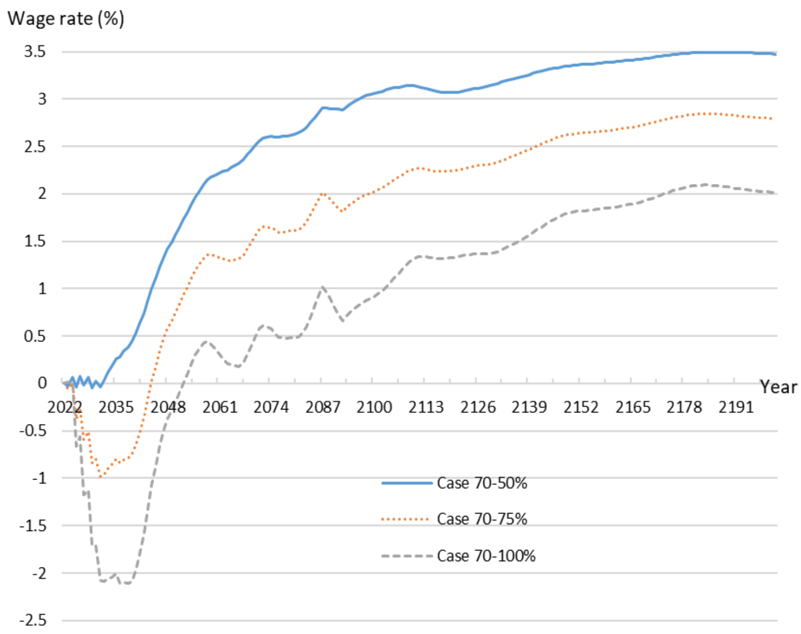

Figure 15 Changes in wage rates for three cases of different employment rates (50%, 75% and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in wage rates from the benchmark case with the 65-starting age for pension benefits.

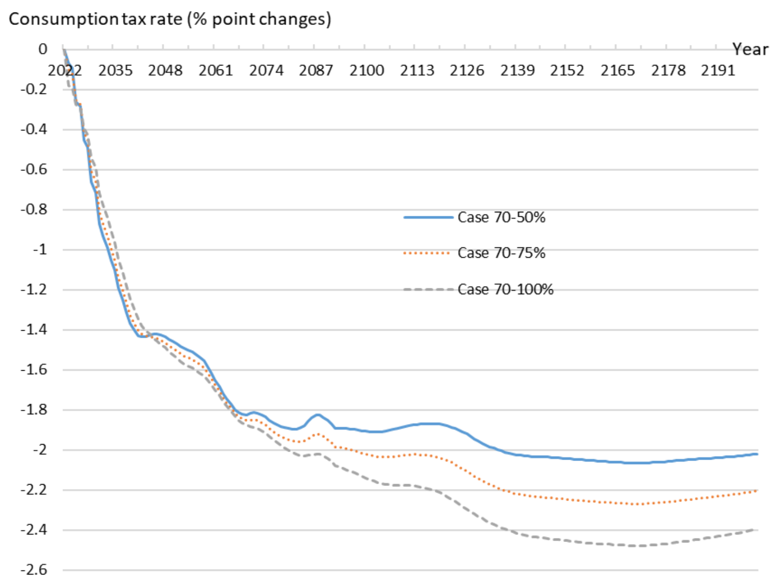

Figure 16 Changes in consumption tax rates for three cases of different employment rates (50%, 75% and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in consumption tax rates from the benchmark case with the 65-starting age for pension benefits.

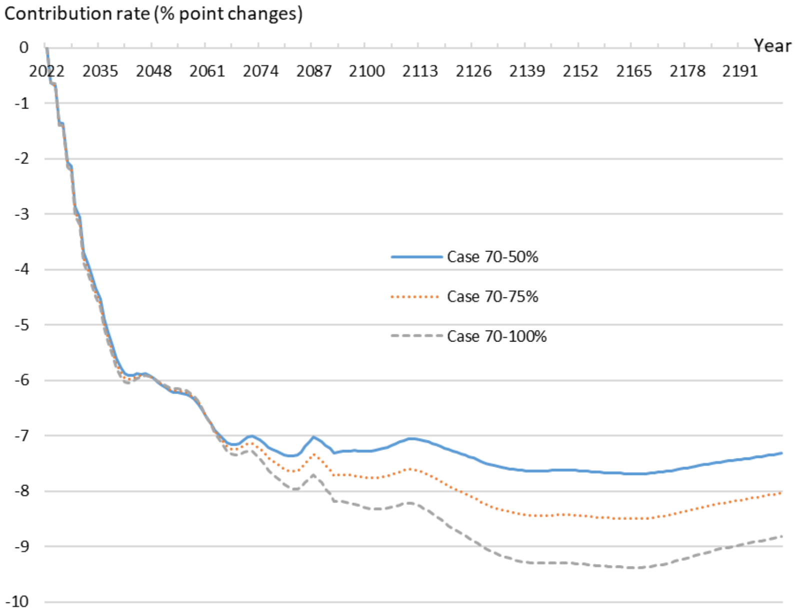

Figure 17 Changes in contribution rates for three cases of different employment rates (50%, 75% and 100%) under the 70-starting age for pension benefits

Notes: The figure shows changes in contribution rates from the benchmark case with the 65-starting age for pension benefits.

Table 1 Normal retirement age of the public pension system and life expectancy

	Normal retirement age	Life expectancy	Difference
Japan	65	84.8	19.8
Germany	67	81.0	14.0
France	67	83.2	16.2
United Kingdom	67	82.1	15.1
United States	67	78.2	11.2
Average (OECD)	67	80.7	13.7

Source: OECD (2023a)

Table 2 Employment rate by age groups in 2022 for male–female totals

Age group	15–24	25–34	35–44	45–54	55–59	60–64	65–69	70–74	75–
Employment rate (%)	46.6	86.6	86.2	86.6	82.8	73.0	50.8	33.5	11.0

Source: Statistics Bureau of Japan (2023b)

Table 3 Simulation cases

Case	Starting age for pension benefits	Employment rate for individuals 65 and above
Benchmark	65	
68-50%	68	50%
68-75%		75%
68-100%		100%
70-50%	70	50%
70-75%		75%
70-100%		100%

Table 4 Exogenous variables for the benchmark simulation

Parameter description	Parameter value	Data source
Share parameter for consumption	φ = 0.5	Nishiyama & Smetters (2005): φ = 0.47
Weight parameter of the number of children to the consumption–leisure composite in utility	$\alpha^{(H)}$=$\alpha^{(U)}$= 0.028148	
Rate of time preference	δ = 0.0001	Oguro et al. (2011): δ = 0.01
Intertemporal substitution elasticity	γ = 0.5	İmrohoroğlu et al. (2017)
Ratio of government subsidies to childrearing costs	ρ = 0.1	Oguro et al. (2011): ρ = 0.1
Ratio of childrearing costs to net lifetime income	β = 0.0385	
Time cost for childrearing	μ =1.7234	
Capital share in production	ε = 0.3794	İmrohoroğlu et al. (2017)
Depreciation rate	δ^k= 0.0821	İmrohoroğlu et al. (2017)
Tax rate on labor income	τ^w= 0.065	Kato (1998): τ^w= 0.065
Tax rate on capital income	τ^r= 0.4	Hayashi & Prescott (2002): τ^r= 0.48; İmrohoroğlu et al. (2017): τ^r= 0.35
Tax rate on inheritance	τ^h= 0.1	Kato (1998): τ^h= 0.1
Ratio of government expenditures to national income	g = 0.1	
Ratio of the part financed by tax transfer to total pension benefit	π = 0.25	Oguro & Takahata (2013): π = 0.25
Replacement ratio for public pension benefits	θ = 0.4	Braun et al. (2009): θ =0.4
Ratio of net public debt to national income	d = 1.5	İmrohoroğlu et al. (2017), Nakajima & Takahashi (2017) : d = 1.3
Compulsory retirement age	RE = 64	
Starting age for receiving public pension benefits	ST = 65	
Ratio of people aged 18 (or 22) and above to the total population	E/Z = 0.84904	
Dependency ratio (i.e., aging rate)	O/Z = 0.31209	

Table 5　Endogenous variables in the 2022 initial steady state

Parameter description	Parameter value
Interest rate, r	0.0722
Wage rate, w	1.0758
Tax rate on consumption, τ^c	0.1308
Contribution rate, τ^p	0.1602
Capital–income ratio, K/Y	2.4595
Total fertility rate (TFR)	1.2600 (low-income class 1.334; high-income class 1.147)
Ratio of net childrearing costs to annual labor income	0.1919 (low-income class) 0.1908 (high-income class)
Ratio of government childcare subsidies to national income, GS/Y	0.0108

Table 6　Population ratios among people with different educational backgrounds

	Population (thousands)	Population share (%)	
Junior high school graduates	689.84	3.19	49.43
High school graduates	10,000.91	46.24	
Technical and junior college	2,146.77	9.93	50.57
University graduates	8,791.18	40.65	
Total (in year 2022)	21,629.35	100	

Source: The Ministry of Health, Labour and Welfare (2023b)

Table 7　Leveled welfare gains for each individual for six scenarios

Case	Welfare gains in yen (million)
68-50%	10.2653
68-75%	13.3978
68-100%	17.2238
70-50%	16.4810
70-75%	20.5372
70-100%	25.3766