Intergenerational Earnings Mobility and Demographic Dynamics:
Welfare Analysis of an Aging Japan

Akira Okamoto (Okayama University)

June 30, 2021

Okayama University Discussion Paper No. I－112

Intergenerational Earnings Mobility and Demographic Dynamics: Welfare Analysis of an Aging Japan[*]

Akira Okamoto[**]
Okayama University

June 30, 2021

Abstract

This paper simulates the effects of intergenerational earnings mobility on individual welfare and future demography in an aging and depopulating Japan. A simulation analysis finds that increased intergenerational mobility across income classes promotes economic growth, and from a long-term perspective, a higher mobility potentially achieves *Pareto improvements.* In the long run, however, it will hinder economic growth. This is because increased mobility increases the population share of individuals with a higher labor productivity, enhancing economic growth in the initial stage, but because of their lower fertility, an increase in their population ratio negatively effects the total population over time.

Keywords: Intergenerational earnings mobility; depopulating and aging societies; demographic dynamics; Pareto improvements; welfare analysis.

JEL classification: H30; C68

---

[*] An earlier version of this paper was presented at the *International Symposium on Business and Economic Studies* held at Okayama University in November in 2019, with the title of "Simulating the Comprehensive Effects of Intergenerational Earnings Mobility in an Aging Japan." I am grateful for the insightful comments and suggestions of Professors Alan J. Auerbach, Ronald D. Lee, and Emmanuel Saez (The University of California, Berkeley). In addition, I want to thank Associate Professor Yoshinari Nomura (Okayama University), who helped me in writing the computer program used for the simulations. Finally, I also wish to acknowledge the financial support from the Social Science Research Council under an Abe Fellowship and the Ministry of Education, Culture, Sports, Science and Technology in Japan (Grant-in-Aid for Scientific Research (C) No. 20K01679).

[**] Professor, Faculty of Economics, Okayama University, Japan. Tsushima-naka, Okayama 700-8530, Japan. *E-mail:* okamot-a@okayama-u.ac.jp

1. Introduction

Japan's population is currently rapidly aging at a speed unprecedented for a developed nation, and simultaneously, the population is decreasing, which has become one of Japan's most important problems. Furthermore, currently, the problem of economic disparities, or income gaps, has become more and more significant. Particularly, it is often pointed out that the parents' inequality is more likely to be passed down to their children. For instance, Hashimoto (2018) and Kikkawa (2018) found that this trend is gradually becoming stronger in Japan, based on the 2015 Nationwide Survey on Social Stratification and Social Mobility (SSM2015). Corak (2016) found in 22 countries that more inequality at a point in time is associated with less generational earnings mobility (see Figure 1, reprinted from Corak 2016). Figure 2 (also excerpted from Corak 2016) shows comparable estimates of the intergenerational elasticity between father and son earnings for 22 countries. Figure 2 indicates that Japan has modest income inequalities and intermediate intergenerational earnings mobility (the intergenerational elasticity between father and son earnings is 0.34).[1]

As illustrated in Figures 1 and 2, most advanced countries, such as the United States (US) and the United Kingdom (UK), face widening income inequalities and increasing disparities, except for Scandinavia. In an aging Japan, it is expected that elderly poverty will become a more serious problem in the near future because cohort income disparity increases with age.[2] For example, the US is an advanced country and known for its significant income inequality and wherein the top income class with a very small population ratio currently takes an extremely large share of the overall income and wealth while there also exists a serious poverty problem.[3]

The problem of economic disparities, or income gaps, will become even more important in Japan for four reasons. First, with a rapidly aging and shrinking population, future high economic

---

1 Ueda (2009) estimated the intergenerational mobility of economic status in Japan, using predicted parental income. The intergenerational elasticity is estimated at 0.41–0.46 for married sons and 0.30–0.38 for married daughters.

2 Ohtake (2003) suggested that income inequality in Japan has been recently expanding and that countermeasures must be implemented especially for younger generations. Ohtake (2003) also found that, compared with the US, Japan has a lower mobility across income classes and a lower possibility of "reversal," resulting in a larger gap in lifetime income.

3 According Stebbins and Comen (2020), the top 1% of earners in the United States account for about 20% of the country's total income annually. Meanwhile, the lowest-earning quarter of Americans account for just 3.7% of income every year.

growth is no longer expected. Since increases in the whole pie cannot be expected in a society with low economic growth, the problem of income redistribution becomes more important because of a limited total income. Second, as reported by the Organization for Economic Co-operation and Development (OECD) (2020), Japan's public expenditure on education is the lowest level in the OECD. This suggests that, compared with other developed countries, Japan relies on private education rather than public education. Additionally, non-regular, contingent, and part-time workers are increasing in Japan. Their salary is relatively low, and their life is unstable and precarious. Japan's child poverty rate is considerably high; it is 14.0% (new standards) in 2018 (Ministry of Health, Labour and Welfare 2019). Thus, the so-called 'a chain of the poverty' is gradually becoming a serious social problem. Third, Iwamoto (2019) found the preponderant problem of artificial intelligence (AI) and employment is economic disparity. Since AI replaces high-skilled routine work, medium-skilled workers are taken jobs and drop down to become low-skilled workers. If the total number of low-skilled workers remains unchanged, their wages will decrease and their jobs will become precarious, exacerbating economic disparity. Fourth, Fisher and Bubola (2020) reportedthat COVID-19 has exacerbated economic inequality, because it burdens many people in the lower economic strata, who are more likely to catch the disease and to die from it. Furthermore, even the healthy are likely to suffer loss of income or health care because of quarantine and other measures. This may also hold true for Japan.

Next, we describe our research method. We use the lifecycle general equilibrium simulation model of overlapping generations, developed by Auerbach and Kotlikoff (1983a, 1983b) and similarly applied in Auerbach and Kotlikoff (1987), Seidman (1983), Auerbach et al. (1989), Altig et al. (2001), Homma et al. (1987), Ihori et al. (2006, 2011), and Okamoto (2005, 2010, 2013). Okamoto (2020) extended the simulation model to introduce the number of children freely chosen by households, thus incorporating endogenous fertility and future demographic dynamics.

Because our paper examines the quantitative effects of changes in intergenerational mobility in Japan across income classes, the mobility between parents and children must be introduced into the model. Therefore, from Okamoto (2020), we incorporated the mobility probability into the simulation model with endogenous fertility, enabling us to analyze intergenerational earnings mobility, which is our main contribution to the literature. Since this model extension is performed in a model framework with endogenous fertility, it can also analyze the quantitative effects of changes in intergenerational income class mobility on the transition of the population of each income class. This extended simulation model is a useful analytical tool because it can rigorously examine the quantitative effects of changes in intergenerational income class mobility from parents to their children on individual welfare and demographic dynamics. Thus, this model allows for a discussion of countermeasures for the impending problems concerning widening income disparities and income class stratification in a depopulating and aging Japan.

Therefore, we introduce the descendent income inequality from parents to their children into the simulation model with endogenous fertility and specifically incorporate two representative households, low- and high-income classes into a cohort. Our model treats intergenerational earnings mobility as intergenerational income class mobility. The model also allows the degree of income class stratification to be freely set or chosen by an exogenously probability matrix, which determines the mobility probability from low-income parents to low- or high-income children and that from high-income parents to low- or high-income children.

Our paper will investigate the quantitative effects of changes in intergenerational income class mobility on individual welfare and future demography, using an extended dynamic simulation model of the Auerbach–Kotlikoff type. First, we extended the simulation model to introduce the number of children into the utility function, which enables future demography to be determined endogenously (Okamoto 2020). Furthermore, into our extended framework with endogenous fertility, we introduced the descendent link between a parent and children and gave the exogenous transition probabilities from the parent's income class to the same (or the other) income class that their children will belong to.

Finally, as shown in Okamoto (2020), our study introduces an additional government institution, the Lump Sum Redistribution Authority (LSRA). Changes in the intergenerational mobility across income classes generally improve the welfare of some generations but reduce that of others. If combined with redistribution from winning to losing generations, such changes may offer the prospect of *Pareto improvements*. Without implementing intergenerational redistribution, however, potential efficiency gains or losses cannot be estimated. Therefore, like Auerbach and Kotlikoff (1987) and Nishiyama and Smetters (2005), we introduce the LSRA as a hypothetical government institution. This distinguishes potential efficiency gains/losses from possible offsetting changes in the welfare of different generations. To isolate pure efficiency gains or losses, we consider simulation cases via LSRA transfers where the probability of intergenerational income class mobility is increased/decreased. The introduction of LSRA transfers enables us to examine policy proposals from a long-term perspective, considering not only the welfare of current generations but of future generations. Because of its ability to quantify alternative policies from a long-term perspective, we will be able to present concrete and useful policy proposals.

The remainder of this paper is organized as follows. Section 2 identifies the basic model applied in the simulation analysis. Section 3 explains the method of simulation analysis and its assumptions. Section 4 evaluates the simulation findings and discusses policy implications. Section 5 summarizes and concludes.

2. Theoretical Framework

We calibrate the simulation of the Japanese economy by applying population data from 2017, estimated by the National Institute of Population and Social Security Research. The model includes 106 overlapping generations, corresponding to ages 0–105 years old. Three types of agents are incorporated: households, firms, and the government. The following subsections describe the basic structures of households, firms, and the government, as well as the market equilibrium conditions.

Our model incorporates intergenerational mobility across income classes based on Kikkawa (2009) who found that Japan's income disparity stems fundamentally from different educational backgrounds between high school and university graduates. On the basis of his study, our model introduces two types of representative agents: the low-income class (i.e., (just) high school graduates) and the high-income class (i.e., university graduates) into a cohort. In this section, we describe the behavior of the low-income class household in the model (see Appendix A for the behavior of the high-income class).

2.1. Household Behavior

The economy is populated by 106 overlapping generations that live with uncertainty, corresponding to ages 0–105. Each agent is assumed to consist of a neutral individual because our model does not distinguish by gender. Each agent enters the economy as a decision-making unit and starts to work at age 18 years, and lives to a maximum age of 105 years. Each household is assumed to consist of one adult and its children. The children aged 0–17 or 0–21 only consume, involving childrearing costs for their parent. Each household faces an age-dependent probability of death. Let $q^t_{j+1|j}$ be the conditional probability that a household born in year t lives from age j to $j+1$. Then the probability of a household born in year t, surviving until s can be expressed by

$$p_s^{t(H)} = \prod_{j=18}^{s-1} q^t_{j+1|j} \,. \tag{1}$$

The probability $q^t_{j+1|j}$ is calculated from data estimated by the National Institute of Population and Social Security Research (2017). Since the survival probability is different among agents with different birth year, agents born in different years have the different utility function.

Each agent who begins its economic life at age 18 chooses perfect-foresight consumption paths (C_s^t), leisure paths (l_s^t), and the number of born children (n_s^t) to maximize a time-separable utility function of the form:

$$U^{t(H)} = \frac{1}{1-\frac{1}{\gamma}}\left[\alpha^{(H)}\sum_{s=18}^{40} p_s^{t(H)}(1+\delta)^{-(s-18)}\left(n_s^{t(H)}\right)^{1-\frac{1}{\gamma}} + (1-\alpha^{(H)})\sum_{s=18}^{105} p_s^{t(H)}(1+\delta)^{-(s-18)}\left\{\left(C_s^{t(H)}\right)^{\phi}\left(l_s^{t(H)}\right)^{1-\phi}\right\}^{1-\frac{1}{\gamma}}\right] \tag{2}$$

This utility function represents the lifetime utility of the agent born in year t. $C_s^{t(H)}$, $l_s^{t(H)}$ and $n_s^{t(H)}$ are respectively consumption, leisure and the number of children to bear (only in the first 23 periods of the life) for an agent born in year t, of age s; $\alpha^{(H)}$ is the utility weight of the number of children relative to the consumption–leisure composite, γ is the intertemporal elasticity of substitution, δ is the adjustment coefficient for discounting the future, and ϕ is the consumption share parameter to leisure.

Fertility choice in the model is only based on the direct utility that households obtain from their offspring, neglecting the investment element of children. The demand for children as *investment goods* played an important role in traditional economies (and still does in developing countries), where transfers from the young to the old arise within the family. In modern advanced countries, however, a pay-as-you-go (PAYG) social security scheme makes the investment aspect of children socialized, as Groezen *et al.* (2003) pointed out. This creates the possibility for households to free-ride on the scheme by rearing fewer or no children, still being entitled to a full pension benefit.

Therefore, we treat children as 'consumption goods' and a parent is assumed to obtain the utility from the number of children born at each age.

As shown in Okamoto (2020), letting $A_s^{t(H)}$ be capital holdings for the agent born in year t, of age s, maximization of Equation (2) is subject to a lifetime budget constraint defined by the sequence:

$$A_{s+1}^{t(H)} = \{1 + r_{t+s}(1-\tau^r)\} A_s^{t(H)} + (1-\tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(H)} \{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\} + a_s^{t(H)} - or_s^{t(H)}$$
$$+ b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) - (1+\tau_{t+s}^c) C_s^{t(H)} - (1-m)(1+\tau_{t+s}^c)\Phi_s^{t(H)} - m(1+\tau_{t+s}^c)\Phi_s^{t(U)}, \quad (3)$$

where r_t is the pretax return to savings, and w_t is the real wage at time t; τ^w, τ^r and τ_t^c are the tax rates on labor income, capital income and consumption, respectively. τ_t^p is the contribution rate to the public pension scheme at time t. All taxes and contributions are collected at the household level. $tc(n^{(H)})$ is the time cost for childrearing. $a^{(H)}$ is the bequest to be inherited, and $or^{(H)}$ is the childrearing cost for orphans. There are no liquidity constraints, and thus the assets $A_s^{(H)}$ can be negative. Terminal wealth must be zero. An individual's earnings ability $e_s^{(H)}$ is an exogenous function of age.

The public pension program is assumed to be a PAYG scheme similar to the current Japanese system. The program starts to collect contributions to the scheme from the age of 20, in accordance with the law. The pension benefit is assumed to comprise only an earnings-related pension:

$$b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) = \begin{cases} \theta H^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) & (s \geq ST) \\ 0 & (s < ST) \end{cases}, \quad (4)$$

where

$$H^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) = \frac{1}{RE-19}\sum_{s=20}^{RE} w_{t+s} e_s^{(H)} \{1 - l_s^{t(H)} - tc_s^t(n_u^{t(H)})\}. \quad (5)$$

The age at which a household born in year t starts to receive the public pension benefit is ST, the average annual labor income for the calculation of pension benefit for each agent is $H^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$, and the weight coefficient of the part proportional to $H^{t(H)}$ is θ. The symbol $b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$ signifies that the amount of public pension benefit is a function of the age profile of labor supply, $\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}$.

A parent is assumed to bear children with the upper limit of 40 years old, and expend for them until they become independent of their parent, namely, during the period when children are from zero to 17 or 21 years old. Regarding the childrearing costs, the model takes account of both monetary and time costs. Here, note that the children aged below 18 or 22 years old do not conduct an economic activity independently, and childrearing costs for their parent arise until they become independent of their parent. The financial costs for rearing the children, for the parent born in year t and s years old, are represented by $\Phi_s^{t(H)}$ and $\Phi_s^{t(U)}$, which are the cost for the children who will become high school graduates and university graduates, respectively:

$$\Phi_s^{t(H)} = \begin{cases} \sum_{k=18}^{s} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=18,19,\cdots,35) \\ \sum_{k=s-17}^{s} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=36,37,\cdots,40) \\ \sum_{k=s-17}^{40} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=41,42,\cdots,57) \end{cases}, \tag{6}$$

$$\Phi_s^{t(H)} = 0 \quad (s=58,59,\cdots,105), \tag{7}$$

$$\Phi_s^{t(U)} = \begin{cases} \sum_{k=18}^{s} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=18,19,\cdots,39) \\ \sum_{k=s-21}^{40} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s=40,41,\cdots,61) \end{cases}, \tag{8}$$

$$\Phi_s^{t(U)} = 0 \quad (s=62,63,\cdots,105), \tag{9}$$

$$\xi^{t(H)} = \beta NW^{t(H)}, \tag{10}$$

where $\xi^{t(H)}$ is the childrearing cost for the parent born in year t, ρ is the rate of government subsidy (including child allowances) to childrearing costs, and β is the ratio of childrearing costs to the net lifetime income, $NW^{t(H)}$, for the parent born in year t.

The children who will become university graduates needs more monetary cost than the children who will become high school graduates simply by the extra four-year (18–21) cost before the independence from their parents. The mobility m denotes the probability in which the children will belong to the high-income class (i.e., university graduates) different from their parent, and $1-m$ is the probability in which they will belong to the low-income class (i.e., high school graduates) same as their parent. The number of children affects the whole available time for a parent, because of the time required for childrearing. The time cost for rearing the children for the parent born in year t, of age s, is represented by

$$tc_s^{t(H)} = \mu n_s^{t(H)}, \tag{11}$$

where μ is the parameter that shows the relation between the number of children and the time required for childrearing, which is simply assumed to be proportional to the number of born children. The time cost is assumed to be same across the two types of children who will become high school graduates or university graduates.

The model contains accidental bequests that result from uncertainty over length of life. The bequests, which comprise assets previously held by deceased households, are distributed equally among all surviving low-income class households at time t. When $BQ_t^{(H)}$ is the sum of bequests inherited by the low-income class households at time t, the bequest to be inherited by each low-income household is defined by

$$a_s^{t(H)} = \frac{(1-\tau^h)BQ_{t+s}^{(H)}}{E_{t+s}^{(H)}}, \tag{12}$$

where

$$BQ_t^{(H)} = \sum_{s=18}^{105}(N_s^{t-s-1(H)} - N_{s+1}^{t-s-1(H)})A_{s+1}^{t-s-1(H)}. \tag{13}$$

τ^h is the tax rate on inheritances of bequests. The amount of inheritances received is linked to the age profile of assets for each household. $E_t^{(H)}$ is the number of the low-income class households conducting an economic activity independently, aged 18 and older. The number of the generation with age s years born in year t is represented by

$$N_s^{t(H)} = p_s^{t(H)}N_0^{t(H)}. \tag{14}$$

Total childrearing cost of the orphans, who are generated as a consequence of parents' uncertainty over length of life, is distributed equally among all surviving low-income class households at time t. When $OR_t^{(H)}$ is the sum of childrearing costs incurred by the low-income class households at time t, the childrearing cost for orphans for each low-income class household is defined by

$$or_s^{t(H)} = \frac{OR_{t+s}^{(H)}}{E_{t+s}^{(H)}}, \tag{15}$$

where

$$OR_t^{(H)} = (1-m)\sum_{s=18}^{57}(N_{s-1}^{t-s(H)} - N_s^{t-s(H)})\Phi_s^{t-s(H)} + m\sum_{s=18}^{61}(N_{s-1}^{t-s(H)} - N_s^{t-s(H)})\Phi_s^{t-s(U)}. \tag{16}$$

Therefore, the net amount of bequests is represented as $a^{(H)} - or^{(H)}$. When we consider the utility maximization problem over time for each agent, besides the flow budget constraint represented by Equation (3), the following constraint is imposed:

$$\begin{cases} 0 \le l_s^{t(H)} \le 1 - tc_s^t(n_s^{t(H)}) & (18 \le s \le RE) \\ l_s^{t(H)} = 1 & (RE+1 \le s \le 105) \end{cases}. \tag{17}$$

This is a constraint that labor supply is nonnegative, and that each household inevitably retires after passing the compulsory retirement age, RE.

Let us consider the case where each agent maximizes expected lifetime utility under two constraints. Each individual maximizes Equation (2) subject to Equations (3) and (17) (see Appendix B for further details). From the utility maximization problem, the equation expressing the evolution of the number of children over time for each individual is characterized by

$$W_s^{t(H)} = \left(\frac{p_{s-1}^{t(H)}}{p_s^{t(H)}} \right) \left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)} \right] W_{s-1}^{t(H)}, \tag{18}$$

$$W_s^{t(H)} = \frac{\alpha^{(H)} k^{1-\frac{1}{\gamma}} (n_s^{t(H)})^{-\frac{1}{\gamma}}}{(1+\tau_{t+s}^c)\left[(1-m)\sum_{g=0}^{17} \Omega_{s,g}^t \xi^{t(H)}(1-\rho) + m\sum_{g=0}^{21} \Omega_{s,g}^t \xi^{t(H)}(1-\rho) \right]}, \tag{19}$$

where $\Omega_{s,0}^t = 1$ for $g = 0$, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g} \{1 + r_{t+s-1+k}(1-\tau^r)\} \right)^{-1}$.

Similarly, that for the consumption–leisure composite is represented by

$$V_s^{t(H)} = \left(\frac{p_{s-1}^{t(H)}}{p_s^{t(H)}} \right) \left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)} \right] V_{s-1}^{t(H)}, \tag{20}$$

$$V_s^{t(H)} = \frac{(1-\alpha^{(H)})\left\{ (C_s^{t(H)})^{\phi} (l_s^{t(H)})^{1-\phi} \right\}^{-\frac{1}{\gamma}} \phi (C_s^{t(H)})^{\phi-1} (l_s^{t(H)})^{1-\phi}}{1+\tau_t^c}. \tag{21}$$

2.2 Firm Behavior

As shown in Okamoto (2020), the model has a single production sector that is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. Capital is homogeneous and depreciating, while labor differs only in efficiency. All forms of labor are perfectly substitutable. Households with different income classes or different ages, however, supply different amounts of some standard measure per unit of labor input.

The aggregate production technology is the standard Cobb-Douglas form:

$$Y_t = K_t^{\varepsilon} L_t^{1-\varepsilon}, \tag{22}$$

where Y_t is aggregate output (national income), K_t is aggregate capital, L_t is aggregate labor supply measured by the efficiency units, and ε is capital's share in production. Using the property subject to a constant-returns-to-scale production function, we can obtain the following equation:

$$Y_t = (r_t + \delta^k) K_t + w_t L_t, \tag{23}$$

where δ^k is the depreciation rate.

2.3 Government Behavior

As shown in Okamoto (2020), at each time t, the government collects tax revenues and issues debt (D_{t+1}) that it uses to finance government purchases of goods and services (G_t) and interest payments on the inherited stock of debt (D_t). The government sector consists of a narrow government sector and a pension sector, and a portion of revenues is transferred to the public pension sector. The public pension system is assumed to be a simple PAYG style and consists only of earnings-related pension. Pension account expenditure is financed by both contributions and a transfer from the general account.

The budget constraint of the narrower government sector at time t is given by

$$D_{t+1} = (1 + r_t) D_t + G_t - T_t, \tag{24}$$

where G_t is total government spending on goods and services, T_t is total tax revenue from labor income, capital income, consumption and inheritances, and D_t is the net government debt at the beginning of year t. D_t is gross public debt minus the accumulated pension fund because the model abstracts the public pension fund, which is represented as a ratio to national income:

$$D_t = dY_t, \tag{25}$$

where d is the ratio of net public debt to national income.

The public pension system is assumed to be a simple PAYG style. The budget constraint of pension sector at time t is represented by

$$R_t = (1 - \pi) B_t, \tag{26}$$

where R_t is total revenue from contributions to the pension program, B_t is total spending on the pension benefit to generations of age ST and above, and π is the ratio of the part financed by the tax transfer from the general account.

The total government spending on goods and service is defined by

$$G_t = gY_t + \pi B_t + GS_t, \tag{27}$$

G_t includes transfers to the public pension sector (πB_t) and the government subsidies to child rearing (GS_t). The government spending except for the transfers and the subsidies is gY_t, which is assumed to be represented as a constant ratio (g) of national income. The spending is assumed to either generate no utility to households or enter household utility functions in a separable fashion.

The total amount of government subsidies (including child allowances) to the childrearing cost in year t is GS_t:

$$GS_t = GS_t^{(H)} + GS_s^{(U)}, \tag{28}$$

$$GS_t^{(H)} = \rho\left[(1-m)\sum_{s=18}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)} + RC_{s,t}^{c(H)}) + m\sum_{s=18}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)})\right], \tag{29}$$

$$\begin{cases} RC_{s,t}^{a(H)} = \sum_{k=18}^{s} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s=18,19,\cdots,35) \\ RC_{s,t}^{b(H)} = \sum_{k=s-17}^{s} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s=36,38,\cdots,40) \\ RC_{s,t}^{c(H)} = \sum_{k=s-17}^{40} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s=41,42,\cdots,57) \end{cases}, \tag{30}$$

$$\begin{cases} RC_{s,t}^{a(U)} = \sum_{k=18}^{s} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s=18,19,\cdots,39) \\ RC_{s,t}^{b(U)} = \sum_{k=s-21}^{40} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s=40,41,\cdots,61) \end{cases}, \tag{31}$$

where $RC_t^{a(H)}$, $RC_t^{b(H)}$ and $RC_t^{c(H)}$ are monetary costs for childrearing when the children will belong to the low-income class same as their parent, namely, they will become high school graduates, and $RC_t^{a(U)}$ and $RC_t^{b(U)}$ are the costs when the children will belong to the high income class different from their parent, namely, they will become university graduates.

$$GS_t^{(U)} = \rho\left[(1-m)\sum_{s=22}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)} + RC_{s,t}^{c(U)}) + m\sum_{s=22}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)})\right], \tag{29'}$$

$$\begin{cases} RC_{s,t}^{a(U)} = \sum_{k=22}^{s} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s=22,23,\cdots,40) \\ RC_{s,t}^{b(U)} = \sum_{k=22}^{40} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s=41,42,43) \\ RC_{s,t}^{c(U)} = \sum_{k=s-21}^{40} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s=44,45,\cdots,61) \end{cases}, \tag{30'}$$

$$\begin{cases} RC_{s,t}^{a(H)} = \sum_{k=22}^{s} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 22, 23, \cdots, 39) \\ RC_{s,t}^{b(H)} = \sum_{k=s-17}^{40} N_k^{t-s(U)} \xi^{t-s(U)} n_k^{t-s(U)} & (s = 40, 41, \cdots, 57) \end{cases}, \tag{31'}$$

where $RC_t^{a(U)}$, $RC_t^{b(U)}$ and $RC_t^{c(U)}$ are financial costs for childrearing when the parent is 22 to 61 years old. Once the parent becomes 62 years old, the cost does not exist because all children are independent from their parent.

The total spending on the pension benefit to generations of age ST and above is represented by

$$B_t = B_t^{(H)} + B_t^{(U)}, \tag{32}$$

where $B_t^{(H)}$ and $B_t^{(U)}$ are the expenditure for the two income classes:

$$B_t^{(H)} = \sum_{s=ST}^{105} N_s^{t-s(H)} b_s^{t-s(H)}, \qquad B_t^{(U)} = \sum_{s=ST}^{105} N_s^{t-s(U)} b_s^{t-s(U)}. \tag{33}$$

The total revenue from pension contributions and the total tax revenue are represented by

$$R_t = \tau^p w_t L_t, \tag{34}$$

$$T_t = \tau^w w_t L_t + \tau^r r_t AS_t + \tau_t^c AC_t + \tau^h BQ_t, \tag{35}$$

where aggregate assets supplied by households, AS_t, and aggregate consumption, AC_t, are given by

$$AS_t = AS_t^{(H)} + AS_t^{(U)}, \tag{36}$$

$$AC_t = AC_t^{(H)} + AC_t^{(U)}. \tag{37}$$

For the low-income class, aggregate assets supplied by households, $AS_t^{(H)}$, and aggregate consumption, $AC_t^{(H)}$, are given by

$$AS_t^{(H)} = \sum_{s=18}^{105} N_s^{t-s(H)} A_s^{t-s(H)}, \tag{38}$$

$$AC_t^{(H)} = \sum_{s=18}^{105} N_s^{t-s(H)} C_s^{t-s(H)} + (1-m) \sum_{s=18}^{57} (RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)} + RC_{s,t}^{c(H)}) + m \sum_{s=18}^{61} (RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)}), \tag{39}$$

where aggregate consumption consists of adult's consumption (at age 18–105 years old) and children's consumption or cost (at age zero to 17 or 21 years old).

For the high-income class, aggregate assets supplied by households, $AS_t^{(U)}$, and aggregate consumption, $AC_t^{(U)}$, are given by

$$AS_t^{(U)} = \sum_{s=22}^{105} N_s^{t-s(U)} A_s^{t-s(U)} \tag{38'}$$

$$AC_t^{(U)} = \sum_{s=22}^{105} N_s^{t-s(U)} C_s^{t-s(U)} + (1-m)\sum_{s=22}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)} + RC_{s,t}^{c(U)}) + m\sum_{s=22}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)}) \tag{39}$$

where aggregate consumption consists of adult's consumption (at age 22–105 years old) and children's consumption or cost (at age zero to 21 or 17 years old).

The total sum of bequests inherited by the households and the total childrearing cost of the orphans at time t are as follows:

$$BQ_t = BQ_t^{(H)} + BQ_t^{(U)}, \tag{40}$$

$$OR_t = OR_t^{(H)} + OR_t^{(U)}. \tag{41}$$

Total population (i.e., the population aged zero to 105), the population aged 18 or 22 to 105 (i.e., independents financially), and the population aged 65 to 105 (i.e., retirees) in year t are respectively represented by

$$Z_t = Z_t^{(H)} + Z_t^{(U)}, \tag{42}$$

$$E_t = E_t^{(H)} + E_t^{(U)}, \tag{43}$$

$$O_t = O_t^{(H)} + O_t^{(U)}. \tag{44}$$

The aging rate (i.e., the old-age dependency ratio), the ratio of the population aged 65 and above to the total population, is given by O_t / Z_t. For the low-income class, the total population, the population aged 18 to 105, and the population aged 65 to 105 in year t are respectively represented by

$$Z_t^{(H)} = \sum_{k=0}^{105}\sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}, \tag{45}$$

$$E_t^{(H)} = \sum_{k=18}^{105}\sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}, \tag{46}$$

$$O_t^{(H)} = \sum_{k=65}^{105}\sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}. \tag{47}$$

For the high-income class, the total population, the population aged 22 to 105, and the population aged 65 to 105 in year t are respectively represented by

$$Z_t^{(U)} = \sum_{k=0}^{105}\sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}, \tag{45'}$$

$$E_t^{(U)} = \sum_{k=22}^{105}\sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}, \tag{46'}$$

$$O_t^{(U)} = \sum_{k=65}^{105}\sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)} \; . \tag{47'}$$

2.4. Market Equilibrium

Finally, equilibrium conditions for the capital, labor and goods markets are described.

1) Equilibrium condition for the capital market

Because aggregate assets supplied by households equal the sum of real capital and net government debt,

$$AS_t = K_t + D_t \, . \tag{48}$$

2) Equilibrium condition for the labor market

Measured in efficiency units, because aggregate labor demand by firms equals aggregate labor supply by households,

$$L_t = L_t^{(H)} + L_t^{(U)} , \tag{49}$$

where $$L_t^{(H)} = \sum_{s=18}^{RE} N_s^{t-s(H)} e_s^{(H)} \{1 - l_s^{t-s(H)} - tc_s^t (n_s^{t(H)})\} , \tag{50}$$

$$L_t^{(U)} = \sum_{s=22}^{RE} N_s^{t-s(U)} e_s^{(U)} \{1 - l_s^{t-s(U)} - tc_s^t (n_s^{t(U)})\} . \tag{51}$$

3) Equilibrium condition for the goods market

Because aggregate production equals the sum of private consumption, private investment and government expenditure,

$$Y_t = AC_t + \{K_{t+1} - (1-\delta^k) K_t\} + G_t \, . \tag{52}$$

An iterative program is performed to obtain the equilibrium values of the above equations.

3. Simulation Analysis

3.1. Method

The simulation model presented in the previous section is solved fundamentally, given the assumption that households have perfect foresight and correctly anticipate interest, wages, the tax and contribution rates, and other factors. If the tax and social security systems and other elements are determined, then the model can be solved using the Gauss–Seidel method (see Auerbach and Kotlikoff (1987) and Heer and Maußner (2005) for the computation process).

Our study assumes the transitional economy of Japan from the initial steady state in 2017 to the final steady state in 2300. Alternative scenarios on the intergenerational mobility across income classes are assumed to be implemented at the end of 2017. For simplicity, 2017 is set as the starting year, and we simulate the demography and the economy in the following years. For the generations that were alive in 2017 and have survived in 2018, we need to pay attention to their formation of future expectations. In 2018, these generations realized that their previous expectations no longer apply and thus again maximize their remaining lifetime utility given perfect foresight. Based on the ex-post age profiles of the number of children to bear, consumption, and leisure for these generations, we calculated their lifetime utility at 18 and 22 years for the low- and high-income classes, respectively.

The LSRA first transfers to each household affected by the change in the degree of intergenerational earnings mobility just enough resources (possibly a negative amount) to return its expected remaining lifetime utility to its pre-change level in the benchmark simulation. For each household that is alive when a change occurs (at the end of 2017), at its age in 2018, the LSRA makes a lump sum transfer, to return its expected remaining lifetime utility to its pre-change utility level. And then the LSRA makes a leveled and common lump-sum transfer to each future household that enters the economy after a change (from 2018 onward), at its age of 18 or 22 years, to return its expected entire lifetime utility back to its pre-change level.

Note that the net present value of these transfers in 2018 across living and future households will generally not sum to 0. Thus, the LSRA makes an additional lump sum transfer to each future household so that the net present value across all transfers is 0. To illustrate, let us assume that these additional transfers are uniform across all future generations, including the low- and high-income classes. If the transfer is positive, then the change has produced extra resources after the expected remaining lifetime utility of each household has been restored to its pre-change level. In this case, we can interpret that the change has created efficiency gains, i.e., *Pareto improvements*. Conversely, if the transfer is negative, then the change has generated an efficiency loss. Thus, the total net present value of all lump sum transfers to current and future generations sums to 0 in 2018, satisfying the LSRA budget constraint (see Appendix B in Nishiyama and Smetters (2005) for further details).

3.2. Simulation cases

We investigate alternative scenarios of the degree of intergenerational mobility across income classes, along with additional simulation cases that add LSRA transfers. According to Kikkawa (2009), the children of high school-graduate parents have a 70% probability of becoming high school graduates and a 30% probability of becoming university graduates, whereas the children of university-graduate parents have a 70% probability of becoming university graduates and a 30% probability of becoming high school graduates. Therefore, the intergenerational mobility probability m in Japan is 0.3 in the benchmark simulation, indicating that parents and their children are likely to belong to the same income class. There are two reasons: (1) university graduate parents sufficiently understand the merits of graduating from a university, whereas high school graduate parents do not, and (2) university-graduate parents tend to give good education to their children, whereas high school-graduate parents are likely to provide relatively poor education to their children.

The following simulation cases are investigated:

3.2.1. Baseline simulation

In the benchmark simulation, the intergenerational mobility probability from the low-income class parent to the high-income class children or from the high-income class parent to the low-income class children is the standard of 0.3, i.e., the normal transitional probability to reach the same income class between a parent and children is 0.7. The benchmark case simulates the transition of the Japanese economy from 2017 to 2300, in which the standard intergenerational mobility (0.3) across income classes is maintained throughout the entire period.

3.2.2. Changes in intergenerational mobility across income classes

We investigate two scenarios in which the intergenerational mobility probability across income classes increases from 0.3 to 0.5 and reduces to 0 for some reason or other. The former scenario is a completely shuffled case: children's future income class (or earnings) is unrelated to their parents', and thus, the transition of the earnings ability is randomly shuffled between parent and child. In other words, regardless of the parent's income class, half of children will belong to the high-income class, and the other half, to the low-income class. Conversely, the latter scenario is completely stratified: children of the high-income class parent will inevitably belong to the high-income class, whereas children of the low-income class parent will always belong to the low-income class.

3.2.3. Cases with LSRA transfers

To distinguish potential efficiency gains/losses from possibly offsetting changes in the welfare of different generations, we introduce the LSRA into our two alternative simulation cases. The LSRA transfers will produce a leveled and common welfare gain/loss for each future household in both the low- and high-income classes.

3.3. Specification of the parameters

We chose realistic parameter values for the Japanese economy on the basis of the literature (Nishiyama and Smetters (2005), Oguro et al. (2011), Imrohoroglu et al. (2017), and Kitao and Mikoshiba (2020)). Table 1 displays the parameter values assigned in the benchmark simulation, and the data source used in the calibration. We chose parameter values such that the calculated values of the model's endogenous variables approach the actual data values. Table 2 presents the endogenous variables in the 2017 initial steady state.

On the basis of Kikkawa (2009), we assume that the low- and the high-income classes correspond to (just) high school and university graduates, respectively. Kikkawa (2009) found that the population of high school graduates and university graduates is roughly equal today and into the future. We use the model with two representative households. Specifically, the following three parameters are crucial for conducting a realistic simulation (the details of assignment of the three parameters will be described below). First, we need the realistic parameter value on the income difference between the two representative agents, namely, the difference of age profile of earnings. Second, we require a realistic parameter value of the difference in birth rates (or fertility rates) between the two representative agents, namely, the difference of preference on the number of children to bear. Third, we need a realistic population ratio for each agent of the two income classes. Thus, the realistic parameter values on these three important parameters are available if our model incorporates two representative agents with different educational backgrounds, on the basis of Kikkawa (2009, 2018).

3.3.1. Demography

Table 3 indicates the population ratio of individuals with a different educational background in 2017, which is estimated from the Basic Survey on Wage Structure (Chingin Sensasu) by the Ministry of Health, Labour and Welfare (2018). Accordingly, the population share of high school graduates (including junior high school graduates) and university graduates (including technical and junior college graduates) is 45.5% and 54.5%, respectively. Figure 3 illustrates the age–population distribution in 2017 based on data from the Ministry of Internal Affairs and Communications (2018a). Figure 3 also denotes the population of high school and university graduates, respectively, for each age. For the elderly, especially those of advanced age, the number of high school graduates exceeds that of the university graduates, whereas for the young and the middle-aged, it is approximately fifty–fifty. For those who are under 18 or 22 years old and undecided to become high school or university graduates, we assume that their population is the same i.e., fifty–fifty on the basis of Kikkawa (2009).

Next, we describe how we assign parameter values for childrearing since our simulation model incorporates endogenous fertility. The Cabinet Office (2010) indicated the average annual childrearing costs for the first-born child to annual income for each age. Based on the survey in the Cabinet Office (2010), we assigned the parameter value of β (i.e., the ratio of childrearing costs to parental net lifetime income) such that the ratio of the annual net childrearing costs to annual labor income for the individual is, on average, close to 19.3%. Thus, β is assigned 0.0385 (the ratio is 22.5% for the low-income class and 17.9% for the high-income class).

The OECD (2020) presented public spending on family benefits in cash, services, and tax measures as a percentage of GDP in 2015. For Japan, the public spending ratios on family benefits in cash, services, and tax measures to GDP are 0.74%, 0.57%, and 0.30%, respectively,[4] When we assign the value of parameter ρ (government childcare subsidies divided by childrearing cost) to 0.1 in the model, as in Oguro et al. (2011), the ratio of the total government subsidies to national income is 1.30% in the initial steady state.

Our model incorporates not only the monetary costs of childrearing but also the time costs. Increases in the number of children diminish a parent's available time because of the time required

[4] In Japan, the ratio of total family benefits to GDP is only 1.61%, whereas it is 2.40% for the 32 OECD member countries, indicating that the level of governmental support to households' childrearing is considerably lower in Japan than in the OECD.

for childrearing. The parameter that makes a connection with the number of children and the time required for childrearing, μ, is proportional to the number of children to rear. This parameter is assigned under the simple assumption that one child needs 1 h per day for childrearing.[5]

Table 4 presents the scheduled number of children for young people aged 21 to 40 in Kikkawa (2018), which is based on a large-scale questionnaire survey (SSM2015). The data were used to assign the parameter values that determine the difference of fertility rates between the two income classes. Table 4 suggests that the scheduled number of children for high school-graduate couples is, on average, 1.14, whereas it is 0.875 for university-graduate couples. The parameter values determining the fertility were chosen so that the total fertility rate (TFR) is 1.43 in the 2017 initial steady state, because Japan's actual TFR was 1.43 in 2017. Consequently, in the initial steady state, the TFR is 1.59 for the low-income class and 1.27 for the high-income class.

3.3.2. Age profile of labor efficiency

The age profiles of earning ability for the two income classes were estimated with data from the Basic Survey on Wage Structure (Chingin Sensasu) by the Ministry of Health, Labour and Welfare for the period (2009–2018). Figure 4 illustrates age–earnings profiles by education. The labor efficiency profiles are constructed from the Japanese data on employment, wages, and monthly work hours.

To estimate the age profiles of earnings ability, $e_s^{(H)}$ and $e_s^{(U)}$, respectively, the following equation is constructed:

$$Q_t = a_0 + a_1 A_t + a_2 A_t^2, \tag{53}$$

where Q is the average monthly cash earnings for high school-graduate workers and university-graduate workers, respectively, and A is the average age for the workers including both males and females. Because bonuses account for a large part of earnings in Japan, Q includes bonuses. For the high school graduates, the starting age of work is earlier (18 years old), but their age profile of earnings is flatter than the university graduates and has a lower level. For the university graduates, the starting age of work is later (22 years old), but their age profile of earnings is steeper with a

[5] Calibrating the value of parameter, μ, that determines the time cost in the model is difficult. In the 2017 initial steady state, an average number of children to which a parent gives birth during the period from 18 or 22 to 40 is 0.0364 per year. We assume that a parent is available time of 16 h per day and that the childrearing time cost for one child is 1 h per day.

higher level.

3.3.3. Taxes and expenditures

Tax rates on labor income, capital income, and inheritances are fixed at the current levels (6.5%, 40%, and 10%, respectively) during the entire period. Tax rates on consumption are endogenously determined to satisfy Equations (24) and (35). General government expenditures, except for transfers to the public pension sector (πB_t), and government subsidies to childrearing (GS_t), are proportional to national income (Y_t) as indicated in Equation (27). The ratio of general expenditure to national income, g, was assigned 0.1 such that the endogenous tax rate on consumption is realistic and plausible in the 2017 initial steady state (i.e., 11.68%). The ratio is held constant at 0.1 during the entire period.

3.3.4. The public pension system

The public pension program is assumed to be a simple PAYG system similar to the current Japanese system. The benefit is assumed to comprise an earnings-related pension, although Japan's actual public pension system is two-tiered: a basic flat pension and an amount proportional to the average annual labor income for each household. General tax revenue finances half of the flat part, whereas contributions to the pension system fund both the remaining half and the entire proportional part. We assigned the ratio (π) of the part financed by the tax transfer from the general account in Equation (26) as 0.25, taken from Oguro and Takahata (2013). The replacement ratio (θ) for public pension benefits in Equation (4) is equal to 40%, following Braun et al. (2009).

The age at which households start to receive public pension benefits (ST) is constant during the entire period. The compulsory retirement age (RE) is the starting age of public pension benefits (ST) minus 1. Thus, after households retire at the end of the year in which they reach compulsory retirement, they immediately start to receive pension benefits.

3.3.5. Government deficits

Net government debt (D_t) is assumed to be proportional to national income to make our simulation feasible. The value of parameter d, which is the ratio of net public debt to national income as given in Equation (25), was assigned based on data from the Ministry of Finance (2018) and the Cabinet Office (2018). After 2017, Japan's national income is expected to decrease as the population declines. Therefore, the assumption that net government debt is proportional to national income during the entire period implies that the government will successfully reduce future government deficits.

3.3.6. Share parameter on consumption in utility

The value of the consumption share parameter, ϕ, in the utility function was assigned based on Altig et al. (2001). Consequently, in the 2017 initial steady state, an individual devotes, on average, approximately 56.2% for the low-income class and 58.4% for the high-income class of the available time endowment (of 16 h per day) to labor during their working years (ages 18–64 or 22–64 years).

3.3.7. Technological progress

The technological progress of private production is significant because it greatly influences economic growth. Thus, careful attention should be paid to our assumptions. Technological progress is assumed to be 0 in the simulation, reflecting Japan's experience during the past two or three decades (see Ihori et al. 2006).

4. Simulation Results

We assume that the baseline simulation has the standard probability (0.3) on the intergenerational mobility across income classes. We analyze two alternative scenarios in which intergenerational mobility is increased to 0.5 and decreased to 0. Furthermore, to isolate the pure efficiency gains or losses from the changes in the mobility probability, we examine additional cases in which the LSRA transfers are introduced in the two alternative cases.

4.1. Baseline simulation

Children of the low-income class parent will belong to the low-income class with the transitional probability of 0.7 and the high-income class with the probability of 0.3. Similarly, children of the high-income class parent will belong to the high-income class with the probability of 0.7 and the low-income class with the probability of 0.3. Figure 5 illustrates the transition of populations for each class in the baseline simulation with the 0.3 mobility probability. In 2017, the low-income class has 76.3 million people, and the high-income class has 50.4 million people. After 2017, the population for each class decreases. Throughout the entire period, the population of the low-income class is greater than the high-income class because the former's fertility rate is higher.

4.2. Changes in the intergenerational mobility probability

First, we discuss the simulation results for the experiment in which the intergenerational mobility probability across income classes is increased to 0.5. In this scenario, the two income classes are randomly shuffled; thus, half the children will belong to the low-income class (or the high-income class), despite their parent's income class. Figure 5 illustrates the transition of each population from 2017 to 2200 for the two income classes, indicating that around 2070 the populations will converge to almost the same level for the two classes, and thereafter, they continue to have the same population. Figures 6 and 7 show changes in the total population and national income, respectively, from the benchmark level. Figure 6 illustrates that an increase in intergenerational mobility results in a lower population over time, compared with the baseline. This is because a high mobility probability increases the number of the high-income class but their fertility is lower than that of the low-income class.

Figure 7 illustrates changes in the national income for the different intergenerational mobility cases, from the level of the benchmark case. An increase in intergenerational mobility across income classes increases national income. This is due to more workers with a high labor productivity (see Figure 5). In the very long run, conversely, an increased mobility probability decreases national income, since after 2134, the negative effect, induced by the decreased total population (see Figure 6), exceeds the positive effect brought about by more workers with a higher earnings ability.

Here, we discuss the reason for the first changes in the level of national income from 2017 to 2060 in Figure 7, for the different intergenerational mobility cases. After 2060, the national income becomes higher in the 0.5 intergenerational mobility scenario than in the benchmark 0.3 case, owing to more workers with a higher labor productivity. From 2017 until 2060, conversely, the national income for the 0.5 mobility case is slightly lower. A possible reason is that increases in high-productivity workers (i.e., university graduates) entail more childrearing costs and negatively impact the economy for a while. In the no intergenerational mobility scenario, the reverse occurs because of relatively more high school graduates.

Next, we discuss the simulation results when the intergenerational mobility probability across income classes is decreased to 0. Figure 5 indicates that after 2017 the population of the low-income class is much larger than that of the high-income class because the fertility rate of the former is higher than that of the latter. The figure indicates that in 2140 the population of the low-income class is almost double that of the high-income class, thereafter the gap is gradually increasing. As indicated in Figure 6, in the no intergenerational mobility scenario, the total population gradually increases over time, compared with that in the benchmark scenario. This is because the zero mobility case has relatively more members of the low-income class with a higher fertility.

This scenario increases workers with a lower labor productivity and thus reduces national income (see Figure 7). In the very long run, conversely, the zero mobility case increases national income because after 2172, the positive effect caused by a larger total population (see Figure 6) exceeds the negative effect induced by more workers with a lower labor productivity. Note that the no intergenerational mobility case achieves a larger population in the very long run, with the population share of the low-income class gradually expanding over time. This may deteriorate per-capita welfare, which will be evaluated in the following subsection, *Cases with LSRA transfers*.

Figures 8 and 9 present the transition of the capital stock and the labor supply, respectively, for the alternate scenarios. The level of capital stock for the 0.5 scenario becomes higher in 2075 but becomes lower after 2127 vis-à-vis the benchmark 0.3 scenario. The labor supply level, measured by efficiency units, for the 0.5 scenario increases in 2057 but decreases after 2138. Increases in the capital stock and the labor supply are mainly caused by more high-earnings-ability workers induced by the randomly shuffled intergenerational income classes. In the very long run, conversely, the negative population effect, caused by the lower fertility of the increased high-income class, exceeds this positive effect. For the zero mobility scenario, the reverse effect can be observed because of relatively more high school graduates: The capital stock level for the zero mobility case decreases after 2070 but increases from 2158, and the labor supply level also decreases in 2055 but increases after 2181.

Figures 10 and 11 illustrate the transition of interest rates and wage rates, respectively, for the two alternate scenarios. In the case of no intergenerational mobility, the population ratio of the low-income class becomes progressively higher over time (see Figure 5). Since high school graduates have a flatter age profile of earnings (see Figure 4), an increase in their population ratio leads to relatively more capital and less labor (measured by efficiency units), which increasingly reduces interest rates and increases wage rates over time. Figure 10 shows that after 2052 the interest rate is lower in the zero mobility case than in the benchmark 0.3 case, whereas Figure 11 shows that after 2052 the wage rate is higher in the zeromobility case than in the 0.3 case. Conversely, for the 0.5 case, the reverse effect occurs (see Figures 10 and 11).

Figure 12 illustrates the transition of consumption tax rates for the two alternative scenarios. The tax rate on consumption is lower until 2117 in the 0.5 mobility case than in the standard 0.3 case, because the former has more workers with a higher labor productivity. In the very long run (from 2118 onward), conversely it increases in the 0.5 case because the population ratio of the high-income class increases over time and their fertility is relatively low, resulting in the population gradually decreasing. A lower ratio of the working population to retirees negatively influences economic growth especially under a PAYG social security system, resulting in higher endogenous tax rates on consumption.

Moreover, as Figure 13 shows, in the very long run (from 2098 onward), the contribution rate to the pension system is also higher in the 0.5 mobility case than in the benchmark 0.3 case. Conversely, the zero mobility scenario has the opposite effect because the population ratio of the low-income class with higher fertility gradually increases and the total population progressively increases over time. A high ratio of the working population to retirees, benefits economic growth particularly under a PAYG social security system, resulting in lower endogenous tax rates on consumption and lower contribution rates in the very long run (see Figures 12 and 13).

4.3. Cases with LSRA transfers

We quantified the effects of changes in the intergenerational mobility across income classes on the macroeconomy and demography. Figure 7 indicates the main results for two alternative scenarios. In this subsection, we focus on the effects of the two scenarios on per-capita welfare. To isolate the efficiency gains or losses from changes in the mobility probability, we introduced LSRA transfers into the two alternate scenarios. Table 5 presents the welfare gains or losses accompanied with changes in the transitional mobility, for each future household in both classes. The high mobility probability case (m = 0.5) generates a per-capita welfare gain of ¥215,040[6]. A high intergenerational mobility increases the population ratio of the workers with a high labor productivity, which improves per-capita welfare. This result implies that increased intergenerational mobility across income classes attains *Pareto improvements*, and thus, the high mobility is desirable from the viewpoint of efficiency. In the very long run, however, in this scenario the total population is substantially reduced, decreasing national income. Conversely, the zero mobility case leads to a per-capita welfare loss of ¥294,450. The absence of the intergenerational mobility across income classes increases the population ratio of the workers with a lower labor productivity, which deteriorates per-capita welfare. Thus, the no mobility scenario is not preferable from the viewpoint of efficiency. In the very long run, however, in this scenario, the total population is significantly increased, resulting in increased national income.

Therefore, the total population effect dominates in the very long run for the two alternative scenarios, and the zero mobility case substantially increases the total population, ultimately increasing economic growth. However, we should note the difference in the population composition of income classes between the two scenarios: As Figure 5 illustrates, in the 0.5 case, the low- and high-income classes have the almost same population share after 2070, whereas in the no mobility case, the population share of the low-income class with a lower labor productivity continues toincrease. The difference in per-capita welfare level between the two cases, reflects the difference in the population structure of income classes.

4.4. Policy implications

Our simulation analysis indicates that a higher intergenerational mobility across income classes generates favorable efficiency results, but in the very long run, it hinders economic growth because of a smaller total population. Hence, we offer this policy implication from a very long-term perspective: A policy that enhances intergenerational earnings mobility should be fundamentally implemented along with measures to reverse the falling birth rate.

---

[6] Japan's GDP in 2017 was calculated at 531.68 trillion yen by the Cabinet Office (2018), and the labor force aged 20–64 years was 57.97 million people in 2017 (Ministry of Internal Affairs and Communications (2018b)). We calculated the real income per worker using these data and also derived the value for GDP in 2017 in our model, yielding a conversion rate between actual amounts of yen and values in the model. Consequently, unity in the model corresponds to 4.75917 million yen.

4.5 Comments

Because the simulation results depend on the model setting and the given parameters, we must be careful about the effects of any parameter changes. The following two parameters are especially crucial. One is the difference in earnings ability between the low- and high-income classes. As their labor productivity gap increases, the labor efficiency effect, caused by changes in the population ratio of workers with different earnings ability, becomes quantitatively greater. The second parameter is the difference in the fertility rate between the two income classes. As this increases, the total population effect, induced by changes in the population ratio of households with different fertility rates, becomes quantitatively larger. Therefore, a large labor productivity gap substantially increases national income in the high intergenerational mobility scenario. Conversely, a large fertility gap greatly enhances the total population effect in the no intergenerational mobility scenario. Since the assignment of these important parameter values is crucial, our study assigned realistic and plausible parameter values using the empirical data.

Next, we offer three challenging tasks for future research. First, our current simulation model treats only the quantity aspect of children and ignores the quality aspect (see Becker and Lewis (1973) for the interaction between children quantity and quality). In reality, however, many Japanese parents are interested in the quality of their children and thus eager for their children's education. In our model, the costs of raising children are a fixed part of the parent's net lifetime income for each of the two income classes, and thus, the absolute level of the expenditure for children is unrelated to child quality. Furthermore, since the net lifetime income for the low-income class is lower than that for the high-income class, the expenditure level on their children is also lower. Because our model assumes intergenerational mobility probabilities across income classes to be exogenous, some children of low-income class parents can become university graduates with a lower cost, whereas some children of the high-income class parents become just high school graduates despite of a higher cost. Especially, in the case of randomly shuffled intergenerational mobility, this setting may be slightly unrealistic.

Therefore, we will introduce parental educational investment into the model in a future paper, under the assumption that education enhances individual human capital and the quality of children. In other words, we will extend the model so that the individual human capital is an increasing function of the education level provided by parents (and the government), which makes it more realistic and plausible. Our current model has exogenous intergenerational mobility probabilities between the low- and high-income classes. However, we will extend the simulation model to include *endogenous* mobility rates induced by the parent's educational investment to their children.

Second, compared with other advanced countries, the public education level in Japan is substantially low, and thus, private education plays a fairly important role. It is well known that the average annual salary for parents of students at the University of Tokyo in Japan is substantially higher than that for general people in their forties and fifties. (This also holds true for Harvard University in the US.) Conversely, recently in Japan, contingent and part-time workers have been increasing. For the poor in the lower economic strata, offering adequate education to their children is difficult. To explore the possible countermeasures, we will analyze the quantitative effects of the promotion of public education by the government. The promotion may help poor children gain a certain level of education and obtain a higher labor productivity, which will equalize the distribution of income and produce more favorable societal outcomes.

Third, although the longevity currently increases for almost all advanced countries, it remains roughly unchanged for the US, mainly because of a short life expectancy for the poor. The National Academies of Sciences/Engineering/Medicine (2015) noted that the gap in life expectancy between rich and poor has recently increased in the US. Specifically, the life expectancy of male workers in the bottom fifth quintile of lifetime earnings does not increase for a period of more than 30 years, whereas for those in the top fifth quintile it substantially increased by more than 7 years. Therefore, in future research we will introduce the life expectancy gap between rich and poor, although the data on the difference in life expectancy between university and high school graduates are not currently available in Japan, as far as we know.

5. Conclusions

Using an extended lifecycle general equilibrium model with endogenous fertility, this study quantified the effects of changes in the intergenerational earnings mobility probability on the demography and individual welfare during the period 2017–2300 in an aging and depopulating Japan. Our simulation results indicate that a high intergenerational mobility across income classes promotes economic growth, and from a long-term perspective, it potentially attains *Pareto improvements*. In this sense, the promotion of intergenerational earnings mobility is desirable. In the very long run, conversely, the promotion reduces economic growth. This is because the increased mobility increases the population ratio of individuals with a higher earnings ability, which promotes economic growth. Because of their relatively low fertility, however, an increase in their population share gradually diminishes the total population. In the very long run, this negative effect exceeds the positive effect induced by more workers with a higher labor productivity.

Conversely, the simulation results indicate that no intergenerational mobility across income classes is relatively harmful to economic growth and from a long-term perspective the absence of the mobility deteriorates per-capita welfare. In this sense, policies which hinder intergenerational earnings mobility are not desirable. This is because the no mobility case increases the population ratio of individuals with a lower labor productivity, which reduce economic growth. In the very long run, conversely, it enhances economic growth because they have a relatively high fertility and an increased population share gradually increases the total population. In the very long run, this positive effect exceeds the negative effect of increased workers with a lower labor productivity.

Appendix A: Model for the High-Income Class (University Graduates)

Here, we describe the household behavior of the high-income class household (i.e., university graduates).

A.1 Household Behavior

Each agent enters the economy as a decision-making unit and starts to work at age 22 years, and lives to a maximum age of 105 years with uncertainty of death. The children aged 0–17 or 0–21 only consume, involving childrearing costs for their parent. The probability of a household born in year t, surviving until s, can be expressed by

$$p_s^{t(U)} = \prod_{j=22}^{s-1} q_{j+1|j}^t . \quad (1)'$$

Each agent who begins its economic life at age 22 chooses perfect-foresight consumption paths ($C_s^{t(U)}$), leisure paths ($l_s^{t(U)}$), and the number of born children ($n_s^{t(U)}$) to maximize a time-separable utility function of the form:

$$U^{t(U)} = \frac{1}{1-\frac{1}{\gamma}}\left[\alpha^{(U)}\sum_{s=22}^{40} p_s^{t(U)}(1+\delta)^{-(s-22)}\left(n_s^{t(U)}\right)^{1-\frac{1}{\gamma}} + (1-\alpha^{(U)})\sum_{s=22}^{105} p_s^{t(U)}(1+\delta)^{-(s-22)}\left\{\left(C_s^{t(U)}\right)^{\phi}\left(l_s^{t(U)}\right)^{1-\phi}\right\}^{1-\frac{1}{\gamma}}\right] \tag{2'}$$

where $C_s^{t(U)}$, $l_s^{t(U)}$ and $n_s^{t(U)}$ are respectively consumption, leisure and the number of children to bear (only in the first 19 periods of the life) for an agent born in year t, of age s. $\alpha^{(U)}$ is the utility weight of the number of children relative to the consumption–leisure composite.

Letting $A_s^{t(U)}$ be capital holdings for the agent born in year t, of age s, maximization of Equation (2)' is subject to a lifetime budget constraint defined by the sequence:

$$A_{s+1}^{t(U)} = \{1 + r_{t+s}(1-\tau^r)\}A_s^{t(U)} + (1-\tau^w - \tau_{t+s}^p)w_{t+s}e_s^{(U)}\{1 - l_s^{t(U)} - tc_s^{t(U)}(n_s^{t(U)})\} + a_s^{t(U)} - or_s^{t(U)}$$
$$+ b_s^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) - (1+\tau_t^c)C_s^{t(U)} - (1-m)(1+\tau_t^c)\Phi_s^{t(U)} - m(1+\tau_t^c)\Phi_s^{t(H)}. \tag{3'}$$

There are no liquidity constraints, and thus the assets can be negative. An individual's earnings ability $e_s^{(U)}$ is an exogenous function of age.

The pension benefit is assumed to comprise only an earnings-related pension:

$$b_s^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) = \begin{cases} \theta H^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) & (s \geq ST) \\ 0 & (s < ST) \end{cases}, \tag{4'}$$

where

$$H^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) = \frac{1}{RE-21}\sum_{s=22}^{RE} w_{t+s}e_s^{(U)}\{1 - l_s^{t(U)} - tc_s^t(n_u^{t(U)})\}. \tag{5'}$$

The average annual labor income for each agent is $H^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)$, and the weight coefficient of the part proportional to $H^{t(U)}$ is θ. The symbol $b_s^{t(U)}\left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)$ in Equation (3)' signifies that the amount of public pension benefit is a function of the age profile of labor supply, $\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}$.

A parent is assumed to bear children and expend for them until they become independent of their parent, namely, during the period when they are from zero to 21 years old. Here, note that the children aged below 22 years old do not conduct an economic activity independently, and only childrearing cost for their parent arises until they become independent of their parent. The financial costs for rearing the children when the parent born in year t is s years old are represented by $\Phi_s^{t(U)}$ and $\Phi_s^{t(H)}$, which are the cost for the children who will become university graduates and high school graduates, respectively:

$$\Phi_s^{t(U)} = \begin{cases} \sum_{k=22}^{s} \xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=22,23,\cdots,40) \\ \sum_{k=22}^{40} \xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=41,42,43) \\ \sum_{k=s-21}^{40} \xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=44,45,\cdots,61) \end{cases}, \tag{6'}$$

$$\Phi_s^{t(U)} = 0 \quad (s=62,63,\cdots,105), \tag{7'}$$

$$\Phi_s^{t(H)} = \begin{cases} \sum_{k=22}^{s} \xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=22,23,\cdots,39) \\ \sum_{k=s-17}^{40} \xi^{t(U)}(1-\rho)n_k^{t(U)} & (s=40,41,\cdots,57) \end{cases}, \tag{8'}$$

$$\Phi_s^{t(H)} = 0 \quad (s=58,59,\cdots,105), \tag{9'}$$

$$\xi^{t(U)} = \beta NW^{t(U)}. \tag{10'}$$

The time cost for rearing the children when the parent born in year t is s years old is represented by

$$tc_s^t = \mu n_s^{t(U)}. \tag{11'}$$

When $BQ_t^{(U)}$ is the sum of bequests inherited by the high income class households at time t, the bequest to be inherited by each high income class household is defined by

$$a_s^{t(U)} = \frac{(1-\tau^h)BQ_{t+s}^{(U)}}{E_{t+s}^{(U)}}, \tag{12'}$$

where $E_t^{(U)}$ is the number of the high income class households conducting an economic activity independently, aged 22 and above, and

$$BQ_t^{(U)} = \sum_{s=22}^{105}(N_s^{t-s-1(U))} - N_{s+1}^{t-s-1(U)})A_{s+1}^{t-s-1(U)}. \tag{13'}$$

The number of the generation born in year t, of age s, is represented by

$$N_s^{t(U)} = p_s^{t(U)} N_0^{t(U)}. \tag{14'}$$

When $OR_t^{(U)}$ is the sum of childrearing costs incurred by the high income class households at time t, the childrearing cost for orphans for each high income class household is defined by

$$or_s^{t(U)} = \frac{OR_{t+s}^{(U)}}{E_{t+s}^{(U)}}, \tag{15'}$$

where

$$OR_t^{(U)} = (1-m)\sum_{s=22}^{61}(N_{s-1}^{t-s(U)} - N_s^{t-s(U)})\Phi_s^{t-s(U)} + m\sum_{s=22}^{57}(N_{s-1}^{t-s(U)} - N_s^{t-s(U)})\Phi_s^{t-s(H)}\,. \qquad (16)'$$

When we consider the utility maximization problem over time for each agent, besides the flow budget constraint represented by Equation (3)', the following constraint is imposed:

$$\begin{cases} 0 \le l_s^{t(U)} \le 1 - tc_s^t(n_s^{t(U)}) & (22 \le s \le RE) \\ l_s^{t(U)} = 1 & (RE+1 \le s \le 105) \end{cases}. \qquad (17)'$$

Each individual maximizes Equation (2)' subject to Equations (3)' and (17)' (see Appendix C for further details). From the utility maximization problem, the equation expressing the evolution of the number of children over time for each individual is characterized by

$$W_s^{t(U)} = \left(\frac{p_{s-1}^{t(U)}}{p_s^{t(U)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right]W_{s-1}^{t(U)}\,, \qquad (18)'$$

$$W_s^{t(U)} = \frac{\alpha^{(U)} k^{1-\frac{1}{\gamma}} (n_s^{t(U)})^{-\frac{1}{\gamma}}}{(1+\tau_{t+s}^c)\left[(1-m)\sum_{g=0}^{21}\Omega_{s,g}^t \xi^{t(U)}(1-\rho) + m\sum_{g=0}^{17}\Omega_{s,g}^t \xi^{t(U)}(1-\rho)\right]}\,, \qquad (19)'$$

where $\Omega_{s,0}^t = 1$ for $g = 0$, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g}\{1 + r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$.

Similarly, that for the consumption–leisure composite is represented by

$$V_s^{t(U)} = \left(\frac{p_{s-1}^{t(U)}}{p_s^{t(U)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right]V_{s-1}^{t(U)}\,, \qquad (20)'$$

$$V_s^{t(U)} = \frac{(1-\alpha^{(U)})\left\{(C_s^{t(U)})^{\phi}(l_s^{t(U)})^{1-\phi}\right\}^{-\frac{1}{\gamma}} \phi (C_s^{t(U)})^{\phi-1}(l_s^{t(U)})^{1-\phi}}{1+\tau_t^c}\,. \qquad (21)'$$

Appendix B: The Utility Maximization Problem for the Low-Income Class

The utility maximization problem over time for each low-income class household in Section 2 is regarded as the maximization of $U^{t(H)}$ in Equation (2) subject to Equations (3) and (17). Let the Lagrange function be

$$L^{t(H)} = U^{t(H)} + \sum_{s=18}^{105}\lambda_s^{t(H)}\Big[-A_{s+1}^{t(H)} + \{1 + r_{t+s}(1-\tau^r)\}A_s^{t(H)} + [1-\tau^w - \tau_{t+s}^p]w_{t+s}e_s^{(H)}\{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\} + a_s^{t(H)}$$
$$- or_s^{t(H)} + b_s^{t(H)}\left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) - (1+\tau_{t+s}^c)C_s^{t(H)} - (1-m)(1+\tau_{t+s}^c)\Phi_s^{t(H)} - m(1+\tau_{t+s}^c)\Phi_s^{t(U)}\Big]$$

$$+\sum_{s=18}^{RE}\eta_s^{t(H)}\left\{1-l_s^{t(H)}-tc_s^t(n_s^{t(H)})\right\},\tag{B1}$$

where $\lambda_s^{t(H)}$ and $\eta_s^{t(H)}$ represent the Lagrange multiplier for Equations (3) and (17), respectively.

The first-order conditions on the number of children $n_s^{t(H)}$, consumption $C_s^{t(H)}$, leisure $l_s^{t(H)}$, and assets $A_{s+1}^{t(H)}$ for s =18, 19, …, 105 can be expressed by

$$p_s^{t(H)}\alpha^{(H)}(1+\delta)^{-(s-18)}(n_s^{t(H)})^{-\frac{1}{\gamma}}=\lambda_s^{t(H)}\left\{\mu[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(H)}+(1-m)(1+\tau_t^c)\sum_{g=0}^{17}\Omega_{s,g}^t\xi^{t(H)}(1-\rho)\right.$$

$$\left.+m(1+\tau_t^c)\sum_{g=0}^{21}\Omega_{s,g}^t\xi^{t(H)}(1-\rho)\right\}+\mu\sum_{k=ST}^{105}\lambda_k^{t(H)}\frac{\theta w_{t+s}e_s^{(H)}}{RE-19}+\mu\eta_s^{t(H)},\tag{B2}$$

where $\Omega_{s,0}^t$ =1 for g =0, $\Omega_{s,g}^t=\left(\prod_{k=1}^{g}\{1+r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$,

$$p_s^{t(H)}(1-\alpha^{(H)})(1+\delta)^{-(s-18)}\left\{(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{1-\phi}\right\}^{-\frac{1}{\gamma}}\phi(C_s^{t(H)})^{\phi-1}(l_s^{t(H)})^{1-\phi}=\lambda_s^{t(H)}(1+\tau_{t+s}^c),\tag{B3}$$

$$p_s^{t(H)}(1-\alpha^{(H)})(1+\delta)^{-(s-18)}\left\{(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{1-\phi}\right\}^{-\frac{1}{\gamma}}(1-\phi)(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{-\phi}$$

$$=\lambda_s^{t(H)}\left\{(1-\tau^w-\tau_{t+s}^p)w_{t+s}e_s^{(H)}\right\}+\sum_{k=ST}^{105}\lambda_k^{t(H)}\frac{\theta w_{t+s}e_s^{(H)}}{RE-19}+\eta_s^{t(H)}\quad(s\le RE),\tag{B4}$$

$$\lambda_s^{t(H)}=\{1+r_{t+s}(1-\tau^r)\}\lambda_{s+1}^{t(H)},\tag{B5}$$

$$\eta_s^{t(H)}\{1-l_s^{t(H)}-tc_s^t(n_s^{t(H)})\}=0\quad(s\le RE),\tag{B6}$$

$$1-l_s^{t(H)}=0\quad(s>RE),\tag{B7}$$

$$\eta_s^{t(H)}\ge 0.\tag{B8}$$

The combination of Equations (B2) and (B5) produces Equations (18) and (19). If the initial value, $n_{18}^{t(H)}$, is given, the initial value, $W_{18}^{t(H)}$, can be derived from Equation (19). If the value, $W_{18}^{t(H)}$, is specified, the value of each age, $W_s^{t(H)}$, can be derived from Equation (18), which generates the value of each age, $n_s^{t(H)}$. If the value, $n_s^{t(H)}$, is specified, the child rearing cost for lifetime is calculated, which gives the lifetime budget constraint represented by Equation (B10).

The combination of Equations (B3) and (B5) produces Equations (20) and (21). If the initial value, $V_{18}^{t(H)}$, is specified, the value of each age, $V_s^{t(H)}$, can be derived from Equation (20). If $V_s^{t(H)}$ is specified, the values of consumption, $C_s^{t(H)}$, and leisure, $l_s^{t(H)}$, at each age are obtained in the method that follows.

For s=18, 19, …, RE, the combination of Equations (B3) and (B4) yields the following expression:

$$C_s^{t(H)} = \left[\frac{\phi \left\{ (1-\tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(H)} + \sum_{k=ST}^{105} \frac{\lambda_k^{t(H)}}{\lambda_s^{t(H)}} \frac{\theta w_{t+s} e_s^{(H)}}{RE-20} + \frac{\eta_s^{t(H)}}{\lambda_s^{t(H)}} \right\}}{(1-\phi)(1+\tau_{t+s}^c)} \right] l_s^{t(H)}. \tag{B9}$$

If the value of $l_s^{t(H)}$ is given under $\eta_s^{t(H)}$=0, the value of $C_s^{t(H)}$ can be obtained using a numerical method, and then the value of $V_s^{t(H)}$ can be derived from Equation (21). The value of $l_s^{t(H)}$ is chosen so that the value of $V_s^{t(H)}$ obtained in the simulation is the closest to that calculated by evolution from $V_{18}^{t(H)}$ through Equation (20). If the value of $l_s^{t(H)}$ chosen is unity or higher, the value of $C_s^{t(H)}$ is obtained from Equation (21) under $l_s^{t(H)}$=1. If it is less than unity, the value of $C_s^{t(H)}$ is derived from Equation (B9).

For s=RE+1, RE+2, …, 105, the condition of $l_s^{t(H)}$=1 leads to the following equation:

$$V_s^{t(H)} = \frac{(1-\alpha^{(H)})\phi (C_s^{t(H)})^{-\frac{\phi}{\gamma}+\phi-1}}{1+\tau_{t+s}^c}. \tag{21''}$$

The value of $C_s^{t(H)}$ is chosen to satisfy this equation.

From Equation (3) and the terminal condition $A_{18}^{t(H)} = A_{106}^{t(H)}$=0, the lifetime budget constraint for an individual (= $NW^{t(H)}$) is derived:

$$\begin{aligned}
&\sum_{s=18}^{RE} \Psi_s^{t(H)} [1-\tau^w - \tau_{t+s}^p] w_{t+s} e_s^{(H)} \{1 - l_s^{t(H)} - tc_s^t(n_s^{t(H)})\} + \sum_{s=ST}^{105} \Psi_s^{t(H)} b_s^{t(H)} \left(\{1 - l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE} \right) + \sum_{s=18}^{105} \Psi_s^{t(H)} (a_s^{t(H)} - or_s^{t(H)}) \\
&= \sum_{s=18}^{105} \Psi_s^{t(H)} (1+\tau_{t+s}^c) C_s^{t(H)} + (1-m) \sum_{s=18}^{35} \sum_{k=18}^{s} \Psi_s^{t(H)} (1+\tau_{t+s}^c) \xi^{t(H)} (1-\rho) n_k^{t(H)} + (1-m) \sum_{s=36}^{40} \sum_{k=s-17}^{s} \Psi_s^{t(H)} (1+\tau_{t+s}^c) \xi^{t(H)} (1-\rho) n_k^{t(H)} \\
&\quad + (1-m) \sum_{s=41}^{57} \sum_{k=s-17}^{40} \Psi_s^{t(H)} (1+\tau_{t+s}^c) \xi^{t(H)} (1-\rho) n_k^{t(H)} + m \sum_{s=18}^{39} \sum_{k=18}^{s} \Psi_s^{t(H)} (1+\tau_{t+s}^c) \xi^{t(H)} (1-\rho) n_k^{t(H)} \\
&\quad + m \sum_{s=40}^{61} \sum_{k=s-21}^{40} \Psi_s^{t(H)} (1+\tau_{t+s}^c) \xi^{t(H)} (1-\rho) n_k^{t(H)},
\end{aligned} \tag{B10}$$

where $\Psi_{18}^{t(H)}$=1 for s=18, $\Psi_s^{t(H)} = \left(\prod_{u=19}^{s} \{1 + r_{t+u}(1-\tau^r)\} \right)^{-1}$ for s=19, 20, …, 105.

Appendix C: The Utility Maximization Problem for the High-Income Class

The utility maximization problem over time for each high-income class household in Appendix A is regarded as the maximization of $U^{t(U)}$ in Equation (2)' subject to Equations (3)' and (17)'. Let the Lagrange function be

$$L^{t(U)} = U^{t(U)} + \sum_{s=22}^{105} \lambda_s^{t(U)} \Big[- A_{s+1}^{t(U)} + \{1 + r_{t+s}(1-\tau^r)\} A_s^{t(U)} + [1 - \tau^w - \tau_{t+s}^p] w_{t+s} e_s^{(U)} \{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\} + a_s^{t(U)}$$
$$- or_s^{t(U)} + b_s^{t(U)} \big(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE} \big) - (1+\tau_{t+s}^c) C_s^{t(U)} - (1-m)(1+\tau_t^c)\Phi_s^{t(U)} - m(1+\tau_t^c)\Phi_s^{t(H)} \Big]$$
$$+ \sum_{s=22}^{RE} \eta_s^{t(U)} \big\{ 1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)}) \big\}, \qquad \text{(C1)}$$

where $\lambda_s^{t(U)}$ and $\eta_s^{t(U)}$ represent the Lagrange multiplier for Equations (3)' and (17)', respectively.

The first-order conditions on the number of children $n_s^{t(U)}$, consumption $C_s^{t(U)}$, leisure $l_s^{t(U)}$, and assets $A_{s+1}^{t(U)}$ for s =22, 23, …, 105 can be expressed by

$$p_s^{t(U)} \alpha^{(U)} (1+\delta)^{-(s-22)} (n_s^{t(U)})^{-\frac{1}{\gamma}} = \lambda_s^{t(U)} \Bigg\{ \mu [1 - \tau^w - \tau_{t+s}^p] w_{t+s} e_s^{(U)} + (1-m)(1+\tau_t^c) \sum_{g=0}^{21} \Omega_{s,g}^t \xi^{t(U)} (1-\rho)$$
$$+ m(1+\tau_t^c) \sum_{g=0}^{17} \Omega_{s,g}^t \xi^{t(U)} (1-\rho) \Bigg\} + \mu \sum_{k=ST}^{105} \lambda_k^{t(U)} \frac{\theta w_{t+s} e_s^{(U)}}{RE - 21} + \mu \eta_s^{t(U)}, \qquad \text{(C2)}$$

where $\Omega_{s,0}^t$ =1 for g =0, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g} \{1 + r_{t+s-1+k}(1-\tau^r)\} \right)^{-1}$,

$$p_s^{t(U)} (1-\alpha^{(U)})(1+\delta)^{-(s-22)} \big\{ (C_s^{t(U)})^{\phi} (l_s^{t(U)})^{1-\phi} \big\}^{-\frac{1}{\gamma}} \phi (C_s^{t(U)})^{\phi-1} (l_s^{t(U)})^{1-\phi} = \lambda_s^t (1+\tau_{t+s}^c), \qquad \text{(C3)}$$

$$p_s^{t(U)} (1-\alpha^{(U)})(1+\delta)^{-(s-22)} \big\{ (C_s^{t(U)})^{\phi} (l_s^{t(U)})^{1-\phi} \big\}^{-\frac{1}{\gamma}} (1-\phi) (C_s^{t(U)})^{\phi} (l_s^{t(U)})^{-\phi}$$
$$= \lambda_s^{t(U)} \big\{ (1 - \tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(U)} \big\} + \sum_{k=ST}^{105} \lambda_k^{t(U)} \frac{\theta w_{t+s} e_s^{(U)}}{RE - 21} + \eta_s^{t(U)} \qquad (s \le RE), \qquad \text{(C4)}$$

$$\lambda_s^{t(U)} = \{1 + r_{t+s}(1-\tau^r)\} \lambda_{s+1}^{t(U)}, \qquad \text{(C5)}$$

$$\eta_s^{t(U)} \{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\} = 0 \qquad (s \le RE), \qquad \text{(C6)}$$

$$1 - l_s^{t(U)} = 0 \qquad (s > RE), \qquad \text{(C7)}$$

$$\eta_s^{t(U)} \ge 0. \qquad \text{(C8)}$$

The combination of Equations (C2) and (C5) produces Equations (18)' and (19)'. If the initial value,

$n_{22}^{t(U)}$, is given, the initial value, $W_{22}^{t(U)}$, can be derived from Equation (19)'. If the value, $W_{22}^{t(U)}$, is specified, the value of each age, $W_s^{t(U)}$, can be derived from Equation (18)', which generates the value of each age, $n_s^{t(U)}$. If the value, $n_s^{t(U)}$, is specified, the child rearing cost for lifetime is calculated, which gives the lifetime budget constraint represented by Equation (C10).

The combination of Equations (C3) and (C5) produces Equations (20)' and (21)'. If the initial value, $V_{22}^{t(U)}$, is specified, the value of each age, $V_s^{t(U)}$, can be derived from equation (20)'. If $V_s^{t(U)}$ is specified, the values of consumption, $C_s^{t(U)}$, and leisure, $l_s^{t(U)}$, at each age are obtained in the method that follows.

For s=22, 23, …, *RE*, the combination of Equations (C3) and (C4) yields the following expression:

$$C_s^{t(U)} = \left[\frac{\phi \left\{ (1-\tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(U)} + \sum_{k=ST}^{105} \frac{\lambda_k^{t(U)}}{\lambda_s^{t(U)}} \frac{\theta w_{t+s} e_s^{(U)}}{RE-21} + \frac{\eta_s^{t(U)}}{\lambda_s^{t(U)}} \right\}}{(1-\phi)(1+\tau_{t+s}^c)} \right] l_s^{t(U)}. \tag{C9}$$

If the value of $l_s^{t(U)}$ is given under $\eta_s^t = 0$, the value of $C_s^{t(U)}$ can be obtained using a numerical method, and then the value of $V_s^{t(U)}$ can be derived from Equation (21)'. The value of $l_s^{t(U)}$ is chosen so that the value of $V_s^{t(U)}$ obtained in the simulation is the closest to that calculated by evolution from $V_{22}^{t(U)}$ through Equation (20)'. If the value of $l_s^{t(U)}$ chosen is unity or higher, the value of $C_s^{t(U)}$ is obtained from Equation (21)' under $l_s^{t(U)}$=1. If it is less than unity, the value of $C_s^{t(U)}$ is derived from Equation (C9).

For s=*RE*+1, *RE*+2, …, 105, the condition of $l_s^{t(U)}$=1 leads to the following equation:

$$V_s^{t(U)} = \frac{(1-\alpha^{(U)})\phi (C_s^{t(U)})^{-\frac{\phi}{\gamma}+\phi-1}}{1+\tau_{t+s}^c} \tag{21)'''}$$

The value of $C_s^{t(U)}$ is chosen to satisfy this equation.

From Equation (3)' and the terminal condition $A_{22}^{t(U)} = A_{106}^{t(U)}$=0, the lifetime budget constraint for an individual ($=NW^{t(U)}$) is derived:

$$\sum_{s=22}^{RE} \Psi_s^{t(U)} [1-\tau^w - \tau_{t+s}^p] w_{t+s} e_s^{(U)} \{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\} + \sum_{s=ST}^{105} \Psi_s^{t(U)} b_s^{t(U)} \left(\{1 - l_u^{t(U)} - tc_u^t(n_u^{t(U)})\}_{u=22}^{RE} \right) + \sum_{s=22}^{105} \Psi_s^{t(U)} (a_s^{t(U)} - or_s^{t(U)})$$

$$= \sum_{s=22}^{105} \Psi_s^{t(U)} (1+\tau_{t+s}^c) C_s^{t(U)} + (1-m) \sum_{s=22}^{40} \sum_{k=22}^{s} \Psi_s^{t(U)} (1+\tau_{t+s}^c) \xi^{t(U)} (1-\rho) n_k^{t(U)} + (1-m) \sum_{s=41}^{43} \sum_{k=22}^{40} \Psi_s^{t(U)} (1+\tau_{t+s}^c) \xi^{t(U)} (1-\rho) n_k^{t(U)}$$

$$+(1-m)\sum_{s=44}^{61}\sum_{k=s-21}^{40}\Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}+m\sum_{s=22}^{39}\sum_{k=22}^{s}\Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}$$

$$+m\sum_{s=40}^{57}\sum_{k=s-17}^{40}\Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}\ , \qquad \text{(C10)}$$

where $\Psi_{22}^{t(U)}=1$ for $s=22$, $\Psi_s^{t(U)}=\left(\prod_{u=23}^{s}\{1+r_{t+u}(1-\tau^r)\}\right)^{-1}$ for $s=23, 24, \ldots, 105$.

Conflict of Interest: The author has received research grants from the Ministry of Education, Culture, Sports, Science and Technology in Japan (Grant-in-Aid for Scientific Research (C) No. 20K01679). However, the author declare that he has no conflict of interest.

References

Altig, D., A. J. Auerbach, L. J. Kotlikoff, Smetters, K. A., and Walliser, J. (2001) "Simulating Fundamental Tax Reform in the United States," *American Economic Review* 91 (3), 574–595.

Auerbach, A. J. and Kotlikoff, L. J. (1983a) "National Savings, Economic Welfare, and the Structure of Taxation" in Feldstein, M. ed., *Behavioral Simulation Methods in Tax Policy Analysis*, Chicago: University of Chicago Press, 459–493.

Auerbach, A. J. and Kotlikoff, L. J. (1983b) "An Examination of Empirical Tests of Social Security and Savings" in Helpman, E., Razin, A. and Sadka, E. eds., *Social Policy Evaluation: An Economic Perspective*, New York: Academic, 161–179.

Auerbach, A. J. and L. J. Kotlikoff (1987) *Dynamic Fiscal Policy,* Cambridge: Cambridge University Press.

Auerbach, A. J., Kotlikoff, L. J., Hagemann, R. P., and Nicoletti, G. (1989) "The Economic Dynamics of an Aging Population: The Case of Four OECD Countries," *OECD Economic Studies*, 97–130.

Becker, G. S. and Lewis, H. G. (1973) "On the Interaction between the Quantity and Quality of Children," *Journal of Political Economy* 81 (2), S279–S288.

Braun, R. A., Ikeda, D., and Joines, D. H. (2009) "The Saving Rate in Japan: Why It Has Fallen and Why It Will Remain Low," *International Economic Review* 50 (1), 291–321.

Corak, M. (2016) "Inequality from generation to generation: The United States in Comparison," IZA DP No. 9929.

Fisher, M. and E. Bubola (2020) "As Coronavirus Deepens Inequality, Inequality Worsens Its Spread," *The New York Times*, March 15. https://www.nytimes.com/2020/03/15/world/europe/coronavirus-inequality.html

Hashimoto, K. (2018) *The New Japanese Class Society*, Kodansha (in Japanese).

Hayashi, F. and Prescott, E. C. (2002) "The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics* 5 (1), 206–235.

Heer, B. and Maußner, A. (2005) *Dynamic General Equilibrium Modelling*, Springer.

Homma, M., Atoda, N., Iwamoto, Y., and Ohtake, F. (1987) "Pension: Aging Society and Pension Plan" in Hamada, K., Horiuchi, A., and Kuroda, M. eds., *Macroeconomic Analysis of the Japanese Economy*, University of Tokyo Press, 149–175 (in Japanese).

Ihori, T., Kato, R. R., Kawade, M., and Bessho, S. (2006) "Public Debt and Economic Growth in an Aging Japan," in Kaizuka, K. and Krueger, A. O. eds., *Tackling Japan's Fiscal Challenges: Strategies to Cope with High Public Debt and Population Aging*, Palgrave Macmillan, 30–68.

Ihori, T., Kato, R. R., Kawade, M., and Bessho, S. (2011) "Health Insurance Reform and Economic Growth: Simulation analysis in Japan," *Japan and the World Economy* 23 (4) 227–239.

Imrohoroglu, S., S. Kitao and T. Yamada (2017) "Can Guest Workers Solve Japan's Fiscal Problems?," *Economic Inquiry* 55 (3), 1287–1307.

Iwamoto, K. (2019) "Artificial Intelligence (AI) and Future of Employment, Human Resource Development, Work Style," Research Institute of Economy, Trade and Industry (RIETI). https://www.rieti.go.jp/jp/papers/contribution/iwamoto-koichi/27.html

Kato, R. (1998) "Transition to an Aging Japan: Public Pension, Savings, and Capital Taxation," *Journal of the Japanese and International Economies* 12 (3), 204–231.

Kikkawa, T. (2009) *The Society Divided by Educational Backgrounds*, Chikumashobo (in Japanese).

Kikkawa, T. (2018) *The Japanese Fragmentation: Detached Non-university Graduate Youngsters*, Kobunsha (in Japanese).

Kitao, S. and M. Mikoshiba (2020) "Females, the elderly, and also males: Demographic aging and macroeconomy in Japan," *Journal of the Japanese and International Economies* 56, 101064.

National Academies of Sciences/Engineering/Medicine (2015) *The Growing Gap in Life Expectancy by Income: Implications for Federal Programs and Policy Responses*, The National Academies Press, Washington, DC.

Nishiyama, S. and K. Smetters (2005) "Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks," *Journal of Political Economy* 113 (5), 1088–1115.

Oguro, K. and J. Takahata (2013) "Child Benefits and Macroeconomic Simulation Analyses: An Overlapping-Generations Model with Endogenous Fertility," *Public Policy Review* 9 (4), 633–659.

Oguro, K., J. Takahata, and M. Shimasawa (2011) "Child Benefit and Fiscal Burden: OLG Model with Endogenous Fertility," *Modern Economy* 2 (4), 602–613.

Okamoto, A. (2005) "Simulating Fundamental Tax Reforms in an Aging Japan," *Economic Systems*

Research 17 (2), 163-185.

Okamoto, A. (2010) "Public Pension Reform in Japan," *Economic Analysis and Policy* 40 (2), 179-208.

Okamoto, A. (2013) "Welfare Analysis of Pension Reforms in an Ageing Japan," *Japanese Economic Review* 64 (4), 452–483.

Okamoto, A. (2020) "Childcare Allowances and Public Pensions: Welfare and Demographic Effects in an Aging Japan," *The B.E. Journal of Economic Analysis and Policy* 20 (2), Article 20190067.

Ohtake, F. (2003) "Making a Society to be Reversed for Young Generations," *The Nihon Keizai Shimbun*, August 22nd, 31 (in Japanese).

Stebbins, S. and E. Comen (2020) "How much do you need to make to be in the top 1% in every state? Here's the list," *USA Today*, July 1st.

Ueda, A. (2009) "Intergenerational Mobility of Earnings and Income in Japan," *The B.E. Journal of Economic Analysis and Policy* 9 (1), Article 54.

van Groezen, B., T. Leers and L. Meijdam (2003) "Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins," *Journal of Public Economics* 87, 233–251.

Data

Cabinet Office (2010) "The Questionnaire Survey through the Internet for the Child-rearing Cost," Cabinet Office Director-general for Policies on Cohesive Society, Japan (in Japanese). https://www8.cao.go.jp/shoushi/shoushika/research/cyousa21/net_hiyo/mokuji_pdf.html.

Cabinet Office (2018) "National Accounts Statistics: GDP Statistics," https://www.esri.cao.go.jp/jp/sna/menu.html.

Ministry of Finance (2018) "FY2017 Budget," https://www.mof.go.jp/english/budget/budget/index.html.

Ministry of Health, Labour and Welfare (2009–2018) "Basic Survey on Wage Structure (Chingin Sensasu)," https://www.e-stat.go.jp/ (in Japanese).

Ministry of Health, Labour and Welfare (2019) "Comprehensive Survey of Living Conditions 2018," https://www.mhlw.go.jp/toukei/saikin/hw/k-tyosa/k-tyosa19/index.html (in Japanese).

Ministry of Internal Affairs and Communications (2018a) "Estimation of the Population (As of 1st Oct 2017)," http://www.stat.go.jp/data/jinsui/2017np/index.htm (in Japanese).

Ministry of Internal Affairs and Communications (2018b) "Labour force and labour force participation rate by age group: Whole Japan," http://www.stat.go.jp/data/roudou/longtime/03roudou.htm (in Japanese).

National Institute of Population and Social Security Research (2017) *Population Projections for Japan: 2016–2115 (Estimation in April, 2017)*, Japan (in Japanese).

OECD (2020) *OECD Family Database*, OECD, Paris. http://www.oecd.org/social/family/database.htm.

Table 1 Exogenous variables for the benchmark simulation

Parameter description	Parameter value	Data source
Share parameter for consumption	$\phi = 0.5$	Nishiyama & Smetters (2005): $\phi = 0.47$
Weight parameter of the number of children to the consumption–leisure composite in utility	$\alpha^{(H)} = 0.04151$ $\alpha^{(U)} = 0.03203$	
Rate of time preference	$\delta = 0.0001$	Oguro et al. (2011): $\delta = 0.01$
Intertemporal substitution elasticity	$\gamma = 0.5$	Imrohoroglu et al. (2017)
Ratio of government subsidies to childrearing costs	$\rho = 0.1$	Oguro et al. (2011): $\rho = 0.1$
Ratio of childrearing costs to net lifetime income	$\beta = 0.0385$	
Time cost for childrearing	$\mu = 1.7234$	
Capital share in production	$\varepsilon = 0.3794$	Imrohoroglu et al. (2017)
Depreciation rate	$\delta^k = 0.0821$	Imrohoroglu et al. (2017)
Tax rate on labor income	$\tau^w = 0.065$	Kato (1998): $\tau^w = 0.065$
Tax rate on capital income	$\tau^r = 0.4$	Hayashi & Prescott (2002): $\tau^r = 0.48$; Imrohoroglu et al. (2017): $\tau^r = 0.35$
Tax rate on inheritance	$\tau^h = 0.1$	Kato (1998)
Ratio of government expenditures to national income	$g = 0.1$	
Ratio of the part financed by tax transfer to total pension benefit	$\pi = 0.25$	Oguro & Takahata (2013)
Replacement ratio for public pension benefits	$\theta = 0.4$	Braun et al. (2009)
Ratio of net public debt to national income	$d = 1.3$	Imrohoroglu et al. (2017)
Compulsory retirement age	$RE = 64$	
Starting age for receiving public pension benefits	$ST = 65$	
Ratio of people aged 21 and above to the total population	$E / Z = 0.82952$	
Dependency ratio (i.e., aging rate)	$O / Z = 0.27742$	

Table 2　Endogenous variables in the 2017 initial steady state

Parameter description	Parameter value
Interest rate, r	0.0754
Wage rate, w	1.0624
Tax rate on consumption, τ^c	0.1168
Contribution rate, τ^p	0.1305
Capital–income ratio, K/Y	2.4096
Total fertility rate (TFR)	1.4300 (low-income class 1.59; high-income class 1.27)
Ratio of net childrearing costs to annual labor income	0.2246 (low-income class) 0.1785 (high-income class)
Ratio of government childcare subsidies to national income, GS/Y	0.01300

Table 3　Population ratios among people with different educational backgrounds

	Population (thousands)	Population share (%)	
Junior high school graduates	675.03	2.97	45.51
High school graduates	9,665.36	42.54	
Technical and junior college	4,281.24	18.84	54.49
University graduates	8,100.33	35.65	
Total (in year 2017)	22,721.96	100	

Source: The Ministry of Health, Labour and Welfare (2018)

Table 4　Scheduled number of children for young people

	Number of children	Average
Young female non-university graduates	1.32	1.14
Young male non-university graduates	0.96	
Young female university graduates	0.91	0.875
Young male university graduates	0.84	

Source: Kikkawa (2018)

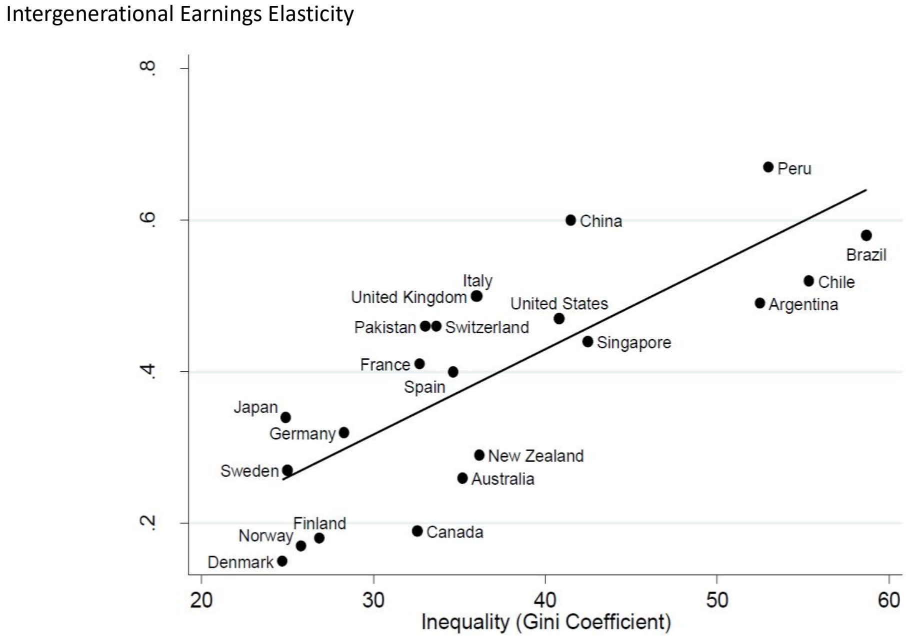
Note: The word “young” denotes the people aged 21 to 40.

Table 5　Welfare gains/losses for each individual for two simulation cases based on intergenerational income class mobility

Intergenerational income class mobility	Welfare gains or losses	Equivalent in yen (thousand)
0.5	0.04518435	215.04
0	−0.06187086	−294.45

Note: The welfare gains/losses for each individual are calculated for changes from the benchmark case (i.e., 0.3 intergenerational income class mobility).

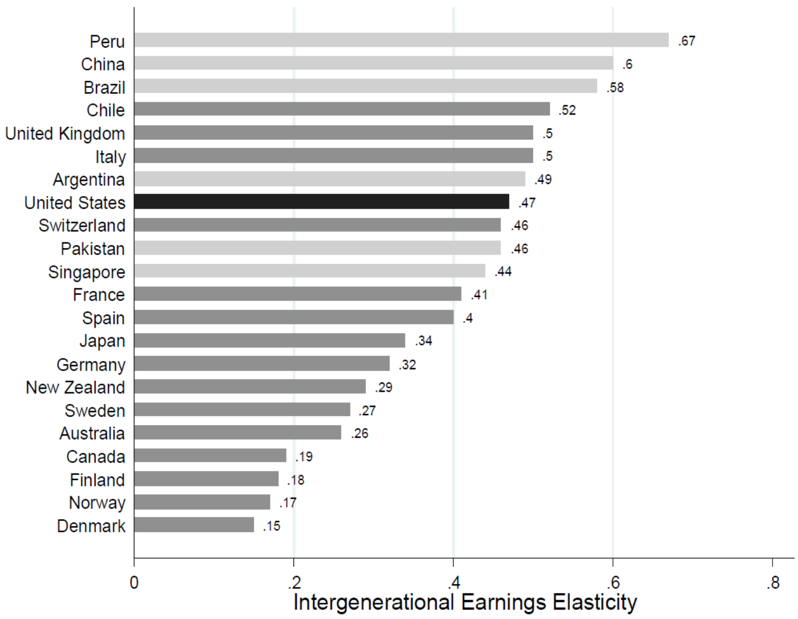
Figure 1 Intergenerational earnings mobility and inequality



Source: Corak (2016)

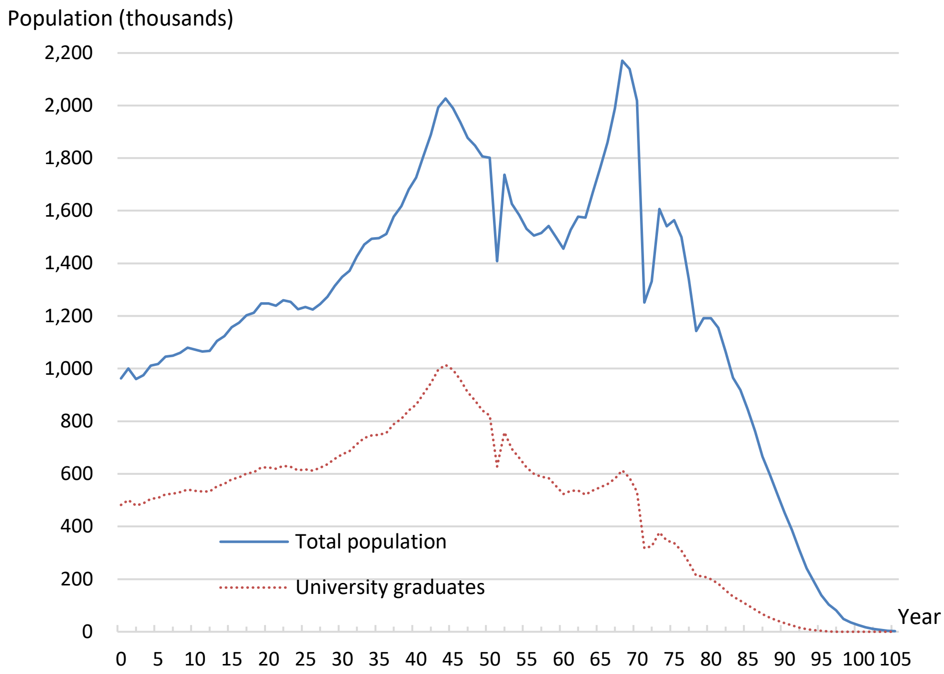
Notes: The vertical axis is the intergenerational elasticity between father and son earnings. The Gini coefficient is obtained from the World Bank. Data points for Italy and the United Kingdom overlap. The upward sloping line is the least squares fitted regression line.

Figure 2 Intergenerational elasticity between father and son earnings



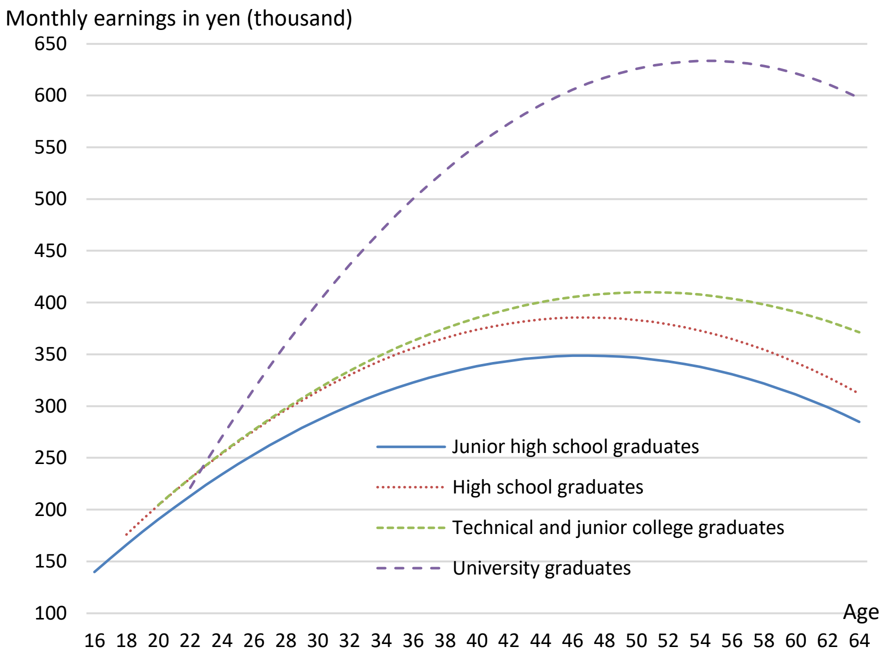
Source: Corak (2016)

Figure 3 Age–population distribution in the 2017 initial steady state



Source: The Ministry of Internal Affairs and Communications (2018a).
Notes: The vertical gap between the total population and the number of university graduates is the number of high school graduates for each age. For young people unsure if they will be (just)high school graduates or university graduates, we assume 50/50.

Figure 4 Age earnings profiles based on educational background



Source: The profiles are estimated from the Ministry of Health, Labour and Welfare (2009–2018).

Figure 5　Transition of population for each income class: Three cases of different intergenerational income class mobility (0, 0.3, 0.5)

Figure 6　Transition of the total population: Two cases of different intergenerational income class mobility (0, 0.5)

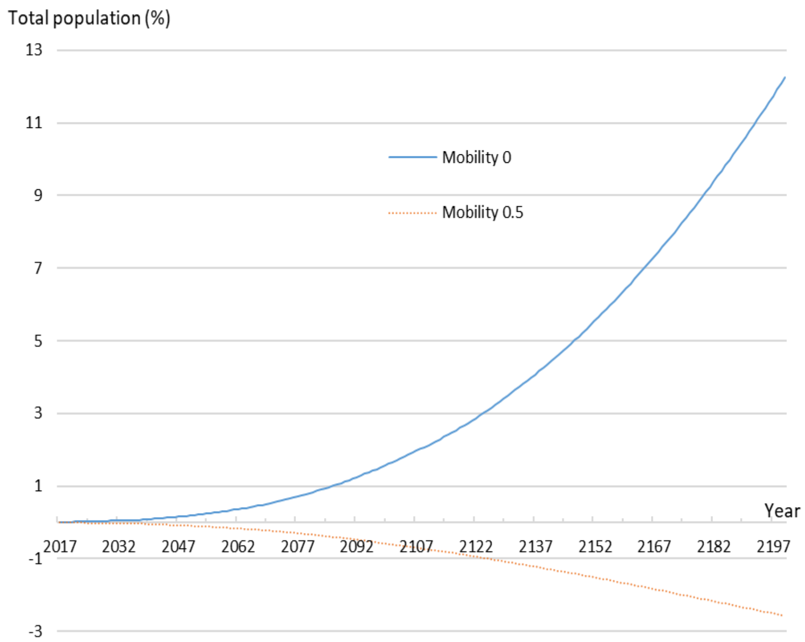


Figure 7 Transition of national income: Two cases of different intergenerational income class mobility (0, 0.5)

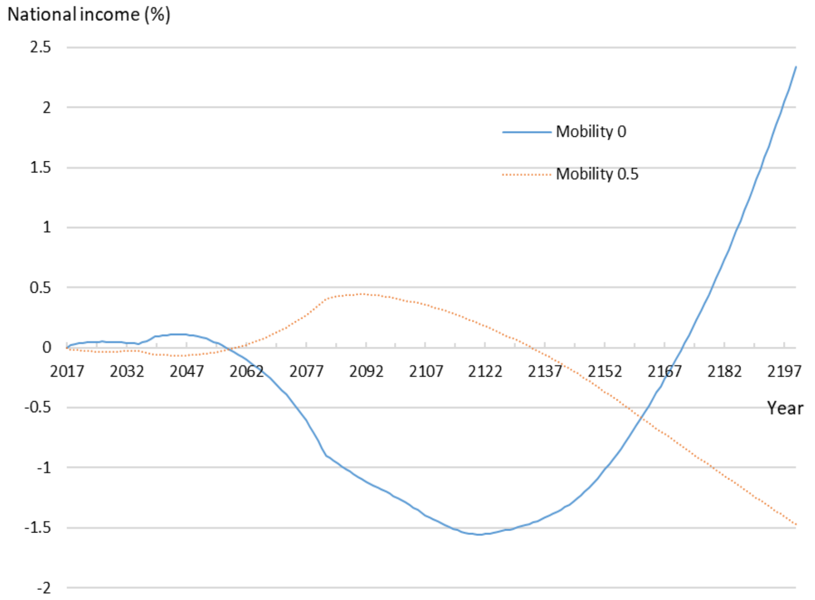


Figure 8 Transition of capital stock: Two cases of different intergenerational income class mobility (0, 0.5)

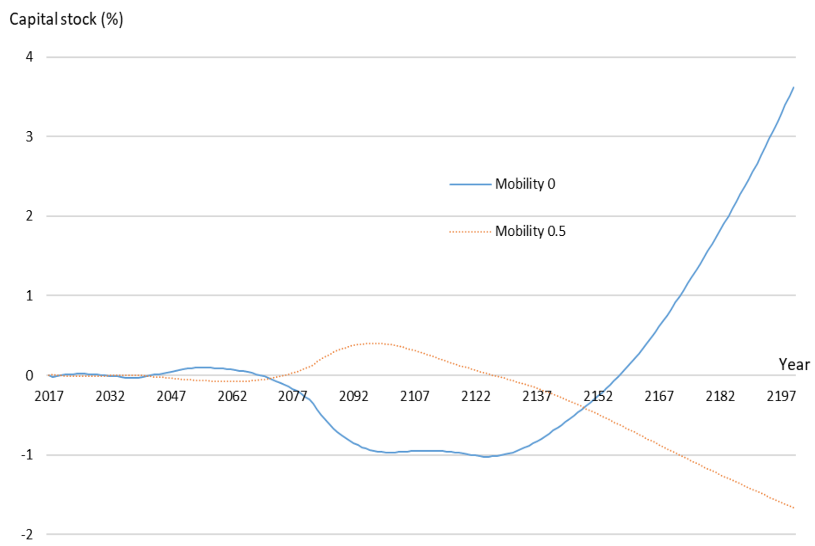


Figure 9 Transition of labor supply: Two cases of different intergenerational income class mobility (0, 0.5)

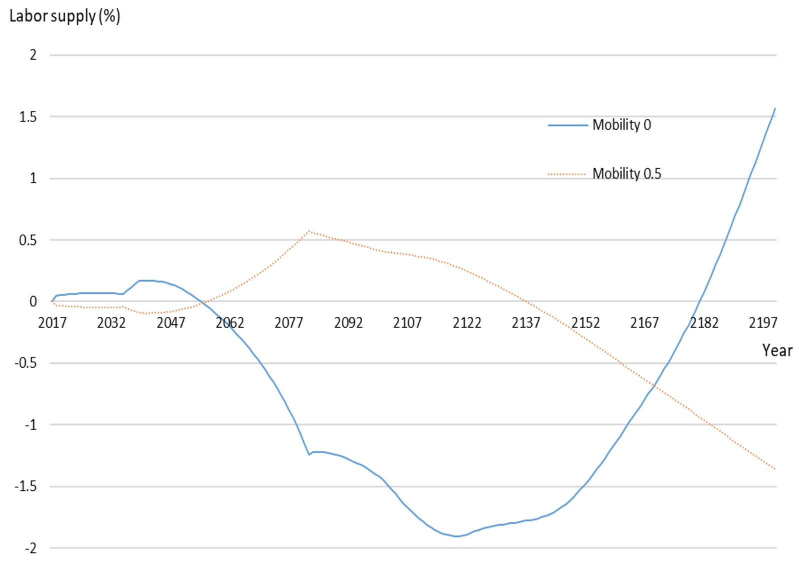


Figure 10 Transition of interest rates: Two cases of different intergenerational income class mobility (0, 0.5)

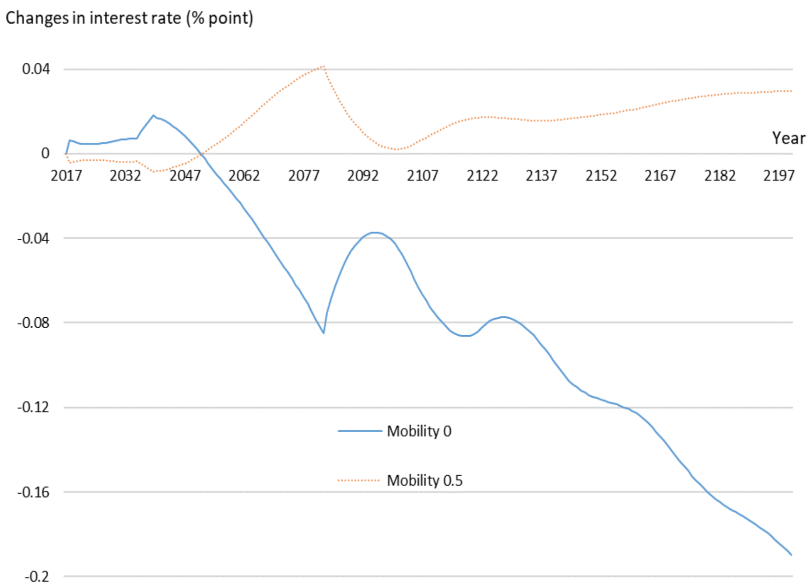


Figure 11 Transition of wage rates: Two cases of different intergenerational income class mobility (0, 0.5)

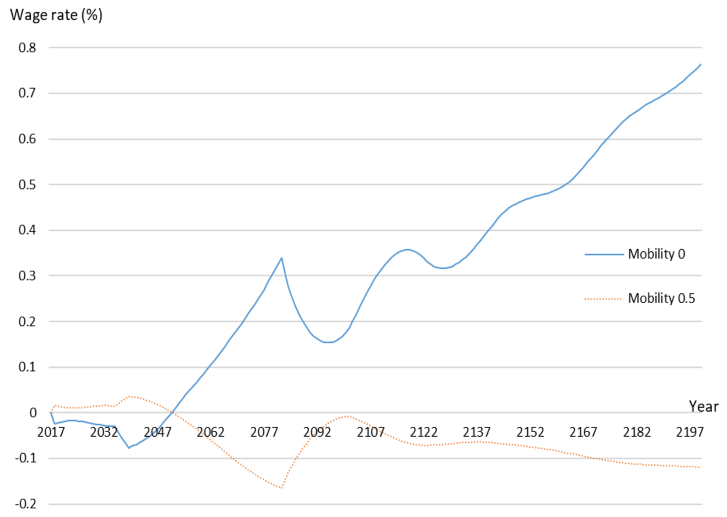


Figure 12 Transition of consumption tax rates: Two cases of different intergenerational income class mobility (0, 0.5)

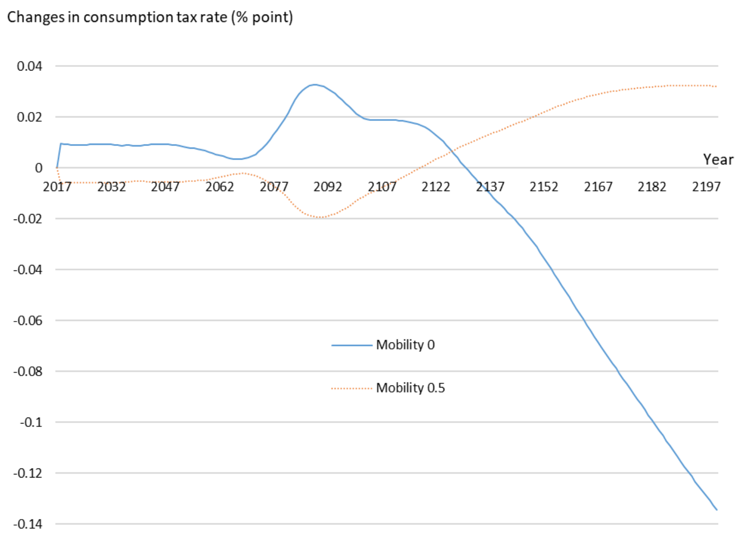


Figure 13 Transition of contribution rates: Two cases of different intergenerational income class mobility (0, 0.5)