Can we resist the population decline in Japan?

Effects of delayed countermeasures and prior announcements

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Abstract

Japan is currently facing a rapid population decline; this paper examines the effects of the countermeasures to falling birthrates and evaluates the long-term impacts on future population and per-capita utility. The paper focuses on analyzing the quantitative effects of a delay in implementing the countermeasures and the impacts of prior announcements of the policy reform. The simulation results reveal that the countermeasures progressively enhance the population and utility; however, the favorable outcomes decrease if the countermeasures are delayed and implemented in 2030 or 2040. If the implementation is delayed to 2050, the population and utility will decline severely. The quantitative effects of advance notice of the reform are substantial. In particular, the effect of notification immediately before the year of implementation (one year in advance) and up to a few years in advance is considerable.

Keywords: Countermeasures to falling birthrates; delayed reforms; prior announcements; welfare; demographic dynamics

JEL classification: H30; C68

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1. Introduction

Based on the results of the 2020 census, the National Institute of Population and Social Security Research (2023) has estimated that Japan's population will fall below 100 million in 2056 and drop to approximately 63 million by 2100. Recognizing that the economic and social system cannot sustain itself if the population continues to decline, a group of experts held a press conference in January 2024 to present their recommendations on population issues. Their proposal indicates that if the population continues its rapid decline, all economic and social systems will be unable to maintain the status quo due to the shrinking market and that many local governments will disappear in regions where the population is declining faster than the rest of the country. The report adds that the total fertility rate (TFR) should be improved to 2.07 in 2060, which is necessary to maintain the population in the long term (1.20 in 2023 in Japan). Furthermore, the public and private sectors should work together to take measures to stabilize the population at 80 million in 2100 and build a society with the capacity for growth.

Thus, the declining birth rates and population are currently among the most pressing issues in Japan. This study's simulation model is well suited for analyzing such issues and can evaluate the countermeasures to falling birthrates from two viewpoints: future demography and individual welfare. We use the lifecycle general equilibrium simulation model of overlapping generations, developed by Auerbach and Kotlikoff (1983a, 1983b) and applied in Auerbach and Kotlikoff (1987), Auerbach et al. (1989), Altig et al. (2001), Homma et al. (1987), Ihori et al. (2006, 2011), and Okamoto (2020, 2021, 2022, 2024a, 2024b).[1] This study uses an extended Auerbach–Kotlikoff dynamic simulation model to investigate the quantitative effects of increases in government childcare subsidies on future population and per-capita welfare.

The simulation model in Okamoto (2020, 2021) introduced the number of children freely chosen by households into the utility function, thus incorporating endogenous fertility and future demographic dynamics. Furthermore, Okamoto (2022, 2024a, 2024b) amalgamated two representative households into a cohort: the low-income class (high school graduates) and the high-income class (university graduates). Okamoto (2022, 2024a, 2024b) introduced the descendent link between parents and children in the extended framework with endogenous fertility. This approach provided the exogenous transition

[1] The lifecycle model is considerably applicable to the Japanese economy. According to Horioka (2021), almost all of the available evidence suggests that the selfish lifecycle model applies, to some extent, in all countries and that there is more consistent support for this model in Japan than in the United States and other countries.

probabilities from the parent's income class to the same (or the other) income class to which their children would belong.

This paper's analytical model is based on Okamoto (2022, 2024a, 2024b). We extended the simulation model to freely set the timing (year) of implementing the policy reform, such as the countermeasures to falling birthrates, and to freely set the timing (year) of prior announcements for implementing the reform. We quantitatively analyzed how increases in the government childcare subsidies in Japan impact the future population levels and the welfare of all generations, including the future and the current generations, for the transition process from 2023 to 2300. Specifically, we focused on analyzing the quantitative effects of a delay in policy reform implementation and prior announcements of the reform on demographic dynamics and per-capita utility. Thus, this paper analyzes the long-run impacts on economic growth, welfare, and population levels for the alternative simulation cases.

Finally, this study introduces an additional government institution, the Lump Sum Redistribution Authority (LSRA). The implementation of the policy reform, such as the countermeasures to falling birthrates, generally improve the welfare of some generations but reduce that of others. If combined with redistribution from winning to losing generations, such changes may offer the prospect of *Pareto improvements*; however, without implementing intergenerational redistribution, potential efficiency gains or losses cannot be estimated. Therefore, like Auerbach and Kotlikoff (1987) and Nishiyama and Smetters (2005), we introduce the LSRA as a hypothetical government institution that distinguishes potential efficiency gains/losses from possible offsetting changes in the welfare of different generations. To isolate pure efficiency gains or losses, we consider simulation cases via LSRA transfers where the government childcare subsidies increase. The introduction of LSRA transfers enables us to examine policy proposals from a long-term perspective, considering the welfare of current and future generations. Because of its ability to quantify alternative policies from a long-term perspective, we can present concrete and valuable policy proposals.

This paper quantifies the effects of the countermeasures to falling birthrates and focuses primarily on analyzing the impacts of a delay in implementing policy reform and the roles of prior announcements. We first examine the projected trend of Japan's declining birthrate and population decline to investigate the abovementioned issue. Next, we address the necessity of countermeasures to falling birthrates in Japan from a theoretical aspect, referring to van Groezen, Leers, and Meijdam (2003) and Aoki and

Vaithianthan (2009). We then mention the related literature (Kitao (2017)), which analyzed the quantitative impacts of a delay in pension reform on the macroeconomy in Japan. Finally, we discuss the related literature (Bütler (1999)), which evaluated the anticipation effects of looming public-pension reforms. Kitao (2017) and Bütler (1999) analyzed pension reforms, in contrast to our study that evaluates the countermeasures to falling birthrates; however, it would be helpful to refer to these studies.

1.1. Demographics in Japan

Japan's population is aging at an unprecedented rate for a developed nation, and the population is simultaneously decreasing, which has become one of the most critical problems. The speed and magnitude of demographic aging in Japan are remarkable, even compared to other advanced countries facing similar challenges; this study's extended lifecycle general equilibrium simulation model with endogenous fertility rigorously reflects such demographic dynamics.

Figure 1 illustrates Japan's population data and projections from 1990 to 2120. Details on the total population until 2023 are based on the actual data from the Statistics Bureau of Japan (2024). Projections after 2023 are based on data (by medium assumptions on fertility and mortality rates) from the National Institute of Population and Social Security Research (2023). The population increased monotonically until 2008, but the trend reversed since 2008 when the number of deaths exceeded births; the total population is projected to continue decreasing throughout the rest of the century. The official projection indicates the population will fall to 62.8 million by 2100, approximately half (50.5%) of the level in 2023 (124.4 million). Low fertility rates and a shrinking number of young females who can bear children are the primary reasons for the ongoing decline in the Japanese population.

Figure 2 shows historical and projected total fertility rates since 1990. The rates first decreased, bottoming at 1.26 in 2005 before somewhat recovering and peaking at 1.45 in 2015. Fertility rates again decreased after 2015, falling to 1.20 in 2023, the lowest since the current population statistics in 1947. These figures indicate an accelerated birthrate decline; however, the National Institute of Population and Social Security Research (2023) projects that the total fertility rate will recover, reaching 1.33 around 2035 and 1.36 by 2070.

Figure 3 denotes historical and projected births and actual marriage results since 2010. First, the number of births in Japan was in a critical situation even before the COVID-19 outbreak, falling significantly below the future population projection (medium projection) by the National Institute of

Population and Social Security Research (2017). The number of births in Japan then declined significantly, partly due to the spread of COVID-19. The number of births fell to 727,277 in 2023, the lowest since the current population statistics in 1899, indicating an accelerated birthrate decline. Moreover, according to the Ministry of Health, Labour and Welfare (2024a), the number of marriages also decreased sharply. The number of marriages declined to 474,717 couples in 2023, the lowest in the postwar period. Because marriages are a prerequisite for childbearing in Japan, the above TFR projection by the National Institute of Population and Social Security Research (2023) may be overly optimistic.

1.2. Countermeasures to falling birthrates

We next take a theoretical perspective to consider the necessity of countermeasures to falling birthrates in Japan. Our model abstracts the investment aspect of children, although children may yield a return in the form of a transfer when their parents become old. In our model, fertility choice is based on the direct utility households obtain from their offspring, neglecting the investment element of children. In other words, our model treats children as *consumption goods* only, neglecting the aspect of *investment goods*. The demand for children as investment goods was vital in traditional economies (and still is in developing countries), where transfers from the young to the old arise within the family.

Conversely, in modern advanced countries, a pay-as-you-go (PAYG) social security scheme makes the investment aspect of children socialized (Groezen et al., 2003). This situation allows households to free-ride on the scheme by rearing fewer or no children while maintaining entitlement to a full pension benefit. Therefore, we treat children as *consumption goods*, and a parent is assumed to obtain the utility from the number of children born at each age. Because parents generally do not consider these positive externalities of children when deciding the number of children, the fertility rate would be lower than the socially optimal value under a PAYG social security scheme. Therefore, childcare allowances are required to correct such external effects caused by a PAYG scheme. Our study quantitatively demonstrated a fact that has only been theoretically verified, contributing to the extant literature.

From the economic welfare viewpoint, Aoki and Vaithianthan (2009) suggested that the crucial questions related to depopulation are whether *market failure* exists in the childbearing decision and whether the private costs and benefits of having children deviate from the social ones. The entitlement to social security benefits during old age does not depend on whether one has children; thus, households are incentivized to free-ride on others' children due to costly childrearing. Under a PAYG pension system, this implies that fertility rates will be lower than socially optimal. If distortions in the economy keep fertility rates below socially optimal levels, government interventions, such as family-oriented policies to the market, are justified. Therefore, in our simulation model, childcare allowances would be effective countermeasures for overcoming *market failure* and thus help improve economic welfare.

1.3. Delay in policy reform implementation

Next, we address the literature on the impacts of a delay in implementing policy reform. Japan is currently facing rapid demographic aging and fiscal challenges; hence, the necessity of aggressive reform for the current PAYG pension system is growing. Kitao (2017) simulated pension reform to reduce the replacement rate of the public pension and raise the retirement age. That study simulated reform that reduced replacement rates by 20%, as embedded in the pension reform of 2004, and gradually raised the normal retirement age from 65 to 68 over 30 years. That study also considered three scenarios with different points in time to initiate reform in 2020, 2030, and 2040, respectively.

The results derived from Kitao (2017) reveal that delaying the reform would suppress economic activities, lower output by up to 4%, and raise the tax burden by more than 8% of total consumption. Delaying reform implies a transfer of costs of demographic aging to the young, which deteriorates the welfare of future generations by up to 3% in terms of consumption equivalence. Delaying reform for a decade or two will generate a sizable and prolonged decline in capital, labor, and economic activities, imposing significantly higher taxes on young and future generations during the transition. The merit of an earlier reform comes at the cost of retirees for whom losses from lower benefits outweigh gains from positive general equilibrium effects. A delay in reform will maintain generous transfers to existing retirees over a longer period, increasing the tax burden for future generations to pay off the accumulated cost of demographic aging.

Therefore, Kitao (2017) demonstrated that a delay in Japan's pension reform deteriorates future generations' welfare, implying a transfer of costs of demographic aging to the young.

1.4. Prior announcements

Finally, we review the literature related to the impacts of prior announcements. Bütler (1999) mentioned the anticipation effects of looming public-pension reforms. That study investigated the implications of looming—but ill-specified—reforms of the current PAYG pension system by conducting several simulations of an artificial economy calibrated to match a Swiss case. Unlike most other contributions dealing with social security reforms, the study focused on the short-run impact of expectations before the reform.

The main conclusions from Bütler (1999) are as follows. Expectations are important and can lead to a substantial fall in consumption and an increase in labor supply well before any reform is implemented. An immediate reduction in consumption reflects the desire to smooth consumption over time in the presence of a negative wealth effect; however, labor supply is also affected by substitution effects, given expected future changes in tax rates. The size of the impact crucially depends on the nature of the expected policy change and the time the pension system's problems are recognized. The sooner the unsustainability of a PAYG system is perceived, the smaller the impact on consumption and labor supply, resulting in smaller welfare losses, both before and after the reform.

The lesson from the simulations in Bütler (1999) is the importance of informed agents. Well-defined reform plans facilitate and improve the individual allocation of resources and, hence, overall welfare. The example of long-horizon expectation patterns in the simulation shows that the more time people are given to adjust, the lower the short-run welfare losses of future pension reform. Most importantly, people should understand what the future structure of a reform pension system might look like, even if the reform's implementation date remains vague initially.

Bütler (1999) demonstrated the critical role of prior announcements of policy reform, showing that prior announcements give people more time to adjust and lower short-run welfare losses of future pension reform.

The remainder of this paper is organized as follows. Section 2 identifies the basic model applied in the simulation analysis, Section 3 explains the method and assumptions of simulation analysis, Section 4 evaluates the simulation findings, and Section 5 summarizes, concludes, and discusses policy implications.

2. Theoretical Framework

We calibrate the simulation of the Japanese economy by applying population data from 2023, estimated by the National Institute of Population and Social Security Research. The model includes 106 overlapping generations, corresponding to ages 0–105 years old. Three types of agents are incorporated: households, firms, and the government. The following subsections describe the basic structures of households, firms, and the government, as well as the market equilibrium conditions.

Our model incorporates intergenerational mobility across income classes based on Kikkawa (2009) who found that Japan's income disparity stems fundamentally from different educational backgrounds between high school and university graduates. On the basis of his study, our model introduces two types of representative agents: the low-income class (i.e., (just) high school graduates) and the high-income class (i.e., university graduates) into a cohort. In this section, we describe the behavior of the low-income class household in the model (see Appendix A for the behavior of the high-income class).

2.1. Household behavior

The economy is populated by 106 overlapping generations that live with uncertainty, corresponding to ages 0–105. Each agent is assumed to consist of a neutral individual because our model does not distinguish by gender. Each agent enters the economy as a decision-making unit and starts to work at age 18 years, and lives to a maximum age of 105 years. Each household is assumed to consist of one adult and its children. The children aged 0–17 or 0–21 only consume, involving childrearing costs for their parent. Each household faces an age-dependent probability of death. Let $q_{j+1|j}^{t}$ be the conditional probability that a household born in year t lives from age j to j +1. Then the probability of a household born in year t, surviving until s can be expressed by

$$p_s^{t(H)} = \prod_{j=18}^{s-1} q_{j+1|j}^{t}. \tag{1}$$

The probability $q_{j+1|j}^{t}$ is calculated from data estimated by the National Institute of Population and Social Security Research (2023). Since the survival probability is different among agents with different birth year, agents born in different years have the different utility function.

Each agent who begins its economic life at age 18 chooses perfect-foresight consumption paths (C_s^t), leisure paths (l_s^t), and the number of born children (n_s^t) to maximize a time-separable utility function of the form:

$$U^{t(H)} = \frac{1}{1-\frac{1}{\gamma}}\left[\alpha^{(H)}\sum_{s=18}^{40} p_s^{t(H)}(1+\delta)^{-(s-18)}\left(n_s^{t(H)}\right)^{1-\frac{1}{\gamma}} + (1 - \alpha^{(H)})\sum_{s=18}^{105} p_s^{t(H)}(1+\delta)^{-(s-18)}\left\{\left(C_s^{t(H)}\right)^{\varphi}\left(l_s^{t(H)}\right)^{1-\varphi}\right\}^{1-\frac{1}{\gamma}}\right]. \tag{2}$$

This utility function represents the lifetime utility of the agent born in year t. $C_s^{t(H)}$, $l_s^{t(H)}$ and $n_s^{t(H)}$ are respectively consumption, leisure and the number of children to bear (only in the first 23 periods of the life) for an agent born in year t, of age s; $\alpha^{(H)}$ is the utility weight of the number of children relative to the consumption–leisure composite, γ is the intertemporal elasticity of substitution, δ is the adjustment coefficient for discounting the future, and φ is the consumption share parameter to leisure.

Letting $A_s^{t(H)}$ be capital holdings for the agent born in year t, of age s, maximization of Equation (2) is subject to a lifetime budget constraint defined by the sequence:

$$A_{s+1}^{t(H)} = \{1 + r_{t+s}(1-\tau^r)\}A_s^{t(H)} + (1-\tau^w-\tau_{t+s}^p)w_{t+s}e_s^{(H)}\{1-l_s^{t(H)}-tc_s^t(n_s^{t(H)})\} + a_s^{t(H)} - or_s^{t(H)} + b_s^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) - (1+\tau_{t+s}^c)C_s^{t(H)} - (1-m)(1+\tau_{t+s}^c)\Phi_s^{t(H)} - m(1+\tau_{t+s}^c)\Phi_s^{t(U)}, \quad (3)$$

where r_t is the pretax return to savings, and w_t is the real wage at time t; τ^w, τ^r and τ_t^c are the tax rates on labor income, capital income and consumption, respectively. τ_t^p is the contribution rate to the public pension scheme at time t. All taxes and contributions are collected at the household level. $tc(n^{(H)})$ is the time cost for childrearing. $a^{(H)}$ is the bequest to be inherited, and $or^{(H)}$ is the childrearing cost for orphans. There are no liquidity constraints, and thus the assets $A_s^{(H)}$ can be negative. Terminal wealth must be zero. An individual's earnings ability $e_s^{(H)}$ is an exogenous function of age.

The public pension program is assumed to be a PAYG scheme similar to the current Japanese system. The program starts to collect contributions to the scheme from the age of 20, in accordance with the law. The pension benefit is assumed to comprise only an earnings-related pension:

$$b_s^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) = \begin{cases} \theta H^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) & (s \geq ST) \\ 0 & (s < ST) \end{cases}, \quad (4)$$

where

$$H^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right) = \frac{1}{RE-19}\sum_{s=20}^{RE} w_{t+s}e_s^{(H)}\left\{1-l_s^{t(H)}-tc_s^t\left(n_s^{t(H)}\right)\right\}. \quad (5)$$

The age at which a household born in year t starts to receive the public pension benefit is ST, the average annual labor income for the calculation of pension benefit for each agent is $H^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$, and the weight coefficient of the part proportional to $H^{t(H)}$ is θ. The symbol $b_s^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$ signifies that the amount of public pension benefit is a function of the age profile of labor supply, $\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}$.

A parent is assumed to bear children with the upper limit of 40 years old, and expend for them until they become independent of their parent, namely, during the period when children are from zero to 17 or

21 years old. Regarding the childrearing costs, the model takes account of both monetary and time costs. Here note that the children aged below 18 or 22 years old do not conduct an economic activity independently, and childrearing costs for their parent arise until they become independent of their parent. The financial costs for rearing the children, for the parent born in year t and s years old, are represented by $\Phi_s^{t(H)}$ and $\Phi_s^{t(U)}$, which are the cost for the children who will become high school graduates and university graduates, respectively:

$$\Phi_s^{t(H)} = \begin{cases} \sum_{k=18}^{s} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s = 18,19,\cdots,35) \\ \sum_{k=s-17}^{s} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s = 36,37,\cdots,40), \\ \sum_{k=s-17}^{40} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s = 41,42,\cdots,57) \end{cases} \tag{6}$$

$$\Phi_s^{t(H)} = 0 \ \ (s = 58,59,\cdots,105), \tag{7}$$

$$\Phi_s^{t(U)} = \begin{cases} \sum_{k=18}^{s} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s = 18,19,\cdots,39) \\ \sum_{k=s-21}^{40} \xi^{t(H)}(1-\rho)n_k^{t(H)} & (s = 40,41,\cdots,61) \end{cases}, \tag{8}$$

$$\Phi_s^{t(U)} = 0 \ \ (s = 62,63,\cdots,105), \tag{9}$$

$$\xi^{t(H)} = \beta NW^{t(H)}, \tag{10}$$

where $\xi^{t(H)}$ is the childrearing cost for the parent born in year t, ρ is the rate of government subsidy (including child allowances) to childrearing costs, and β is the ratio of childrearing costs to the net lifetime income, $NW^{t(H)}$, for the parent born in year t.

The children who will become university graduates needs more monetary cost than the children who will become high school graduates simply by the extra four-year (18–21) cost before the independence from their parents. The mobility m denotes the probability in which the children will belong to the high-income class (i.e., university graduates) different from their parent, and $1 - m$ is the probability in which they will belong to the low-income class (i.e., high school graduates) same as their parent. The number of children affects the whole available time for a parent, because of the time required for childrearing. The time cost for rearing the children for the parent born in year t, of age s, is represented by

$$tc_s^t(n_s^{t(H)}) = \mu n_s^{t(H)}, \tag{11}$$

where μ is the parameter that shows the relation between the number of children and the time required for childrearing, which is simply assumed to be proportional to the number of born children. The time cost is assumed to be same across the two types of children who will become high school graduates or university graduates.

The model contains accidental bequests that result from uncertainty over length of life. The bequests, which comprise assets previously held by deceased households, are distributed equally among all surviving low-income class households at time t. When $BQ_t^{(H)}$ is the sum of bequests inherited by the low-income class households at time t, the bequest to be inherited by each low-income household is defined by

$$a_s^{t(H)} = \frac{(1-\tau^h)BQ_{t+s}^{(H)}}{E_{t+s}^{(H)}}, \tag{12}$$

where

$$BQ_t^{(H)} = \sum_{s=18}^{105}(N_s^{t-s-1(H)} - N_{s+1}^{t-s-1(H)})A_{s+1}^{t-s-1(H)}. \tag{13}$$

τ^h is the tax rate on inheritances of bequests. The amount of inheritances received is linked to the age profile of assets for each household. $E_t^{(H)}$ is the number of the low-income class households conducting an economic activity independently, aged 18 and older. The number of the generation with age s years born in year t is represented by

$$N_s^{t(H)} = p_s^{t(H)} N_0^{t(H)}. \tag{14}$$

Total childrearing cost of the orphans, who are generated as a consequence of parents' uncertainty over length of life, is distributed equally among all surviving low-income class households at time t. When $OR_t^{(H)}$ is the sum of childrearing costs incurred by the low-income class households at time t, the childrearing cost for orphans for each low-income class household is defined by

$$or_s^{t(H)} = \frac{OR_{t+s}^{(H)}}{E_{t+s}^{(H)}}, \tag{15}$$

where

$$OR_t^{(H)} = (1-m)\sum_{s=18}^{57}(N_{s-1}^{t-s(H)} - N_s^{t-s(H)})\Phi_s^{t-s(H)} + m\sum_{s=18}^{61}(N_{s-1}^{t-s(H)} - N_s^{t-s(H)})\Phi_s^{t-s(U)}. \tag{16}$$

Therefore, the net amount of bequests is represented as $a^{(H)} - or^{(H)}$. When we consider the utility maximization problem over time for each agent, besides the flow budget constraint represented by Equation (3), the following constraint is imposed:

$$\begin{cases} 0 \le l_s^{t(H)} \le 1 - tc_s^t(n_s^{t(H)}) (18 \le s \le RE) \\ \qquad l_s^{t(H)} = 1\ (RE+1 \le s \le 105) \end{cases}. \tag{17}$$

This is a constraint that labor supply is nonnegative, and that each household inevitably retires after passing the compulsory retirement age, RE.

Let us consider the case where each agent maximizes expected lifetime utility under two constraints. Each individual maximizes Equation (2) subject to Equations (3) and (17) (see Appendix B for further details). From the utility maximization problem, the equation expressing the evolution of the number of children over time for each individual is characterized by

$$W_s^{t(H)} = \left(\frac{p_{s-1}^{t(H)}}{p_s^{t(H)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right] W_{s-1}^{t(H)}, \tag{18}$$

$$W_s^{t(H)} = \frac{\alpha^{(H)} k^{1-\frac{1}{\gamma}} (n_s^{t(H)})^{-\frac{1}{\gamma}}}{(1+\tau_{t+s}^c)\left[(1-m)\sum_{g=0}^{17} \Omega_{s,g}^t \xi^{t(H)}(1-\rho)+m\sum_{g=0}^{21} \Omega_{s,g}^t \xi^{t(H)}(1-\rho)\right]}, \tag{19}$$

where $\Omega_{s,0}^t$=1 for $g = 0$, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g}\{1 + r_{t+s-1+k}(1 - \tau^r)\}\right)^{-1}$.

Similarly, that for the consumption–leisure composite is represented by

$$V_s^{t(H)} = \left(\frac{p_{s-1}^{t(H)}}{p_s^{t(H)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right] V_{s-1}^{t(H)}, \tag{20}$$

$$V_s^{t(H)} = \frac{(1-\alpha^{(H)})\left\{(C_s^{t(H)})^{\varphi}(l_s^{t(H)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}} \varphi (C_s^{t(H)})^{\varphi-1}(l_s^{t(H)})^{1-\varphi}}{1+\tau_t^c}. \tag{21}$$

2.2. Firm behavior

The model has a single production sector that is assumed to behave competitively using capital and labor, subject to a constant-returns-to-scale production function. Capital is homogeneous and depreciating, while labor differs only in efficiency. All forms of labor are perfectly substitutable. Households with different income classes or different ages, however, supply different amounts of some standard measure per unit of labor input.

The aggregate production technology is the standard Cobb-Douglas form:

$$Y_t = K_t^{\varepsilon} L_t^{1-\varepsilon}, \tag{22}$$

where Y_t is aggregate output (national income), K_t is aggregate capital, L_t is aggregate labor supply measured by the efficiency units, and ε is capital's share in production. Using the property subject to a constant-returns-to-scale production function, we can obtain the following equation:

$$Y_t = (r_t + \delta^k)K_t + w_t L_t, \tag{23}$$

where δ^k is the depreciation rate.

2.3. Government behavior

At each time t, the government collects tax revenues and issues debt (D_{t+1}) that it uses to finance government purchases of goods and services (G_t) and interest payments on the inherited stock of debt (D_t). The government sector consists of a narrow government sector and a pension sector, and a portion of revenues is transferred to the public pension sector. The public pension system is assumed to be a simple PAYG style and consists only of earnings-related pension. Pension account expenditure is financed by both contributions and a transfer from the general account.

The budget constraint of the narrower government sector at time t is given by

$$D_{t+1} = (1 + r_t)D_t + G_t - T_t, \tag{24}$$

where G_t is total government spending on goods and services, T_t is total tax revenue from labor income, capital income, consumption and inheritances, and D_t is the net government debt at the beginning of year t. D_t is gross public debt minus the accumulated pension fund because the model abstracts the public pension fund, which is represented as a ratio to national income:

$$D_t = dY_t, \tag{25}$$

where d is the ratio of net public debt to national income.

The public pension system is assumed to be a simple PAYG style. The budget constraint of pension sector at time t is represented by

$$R_t = (1 - \pi)B_t, \tag{26}$$

where R_t is total revenue from contributions to the pension program, B_t is total spending on the pension benefit to generations of age ST and above, and π is the ratio of the part financed by the tax transfer from the general account.

The total government spending on goods and service is defined by

$$G_t = gY_t + \pi B_t + GS_t, \tag{27}$$

where G_t includes transfers to the public pension sector (πB_t) and the government subsidies to child rearing (GS_t). The government spending except for the transfers and the subsidies is gY_t, which is assumed to be represented as a constant ratio (g) of national income. The spending is assumed to either generate no utility to households or enter household utility functions in a separable fashion.

The total amount of government subsidies (including child allowances) to the childrearing cost in year t is GS_t:

$$GS_t = GS_t^{(H)} + GS_s^{(U)}, \tag{28}$$

$$GS_t^{(H)} = \rho\left[(1-m)\textstyle\sum_{s=18}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)} + RC_{s,t}^{c(H)}) + m\sum_{s=18}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)})\right], \tag{29}$$

$$\begin{cases} RC_{s,t}^{a(H)} = \sum_{k=18}^{s} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s = 18,19,\cdots,35) \\ RC_{s,t}^{b(H)} = \sum_{k=s-17}^{s} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s = 36,38,\cdots,40), \\ RC_{s,t}^{c(H)} = \sum_{k=s-17}^{40} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s = 41,42,\cdots,57) \end{cases} \tag{30}$$

$$\begin{cases} RC_{s,t}^{a(U)} = \sum_{k=18}^{s} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s = 18,19,\cdots,39) \\ RC_{s,t}^{b(U)} = \sum_{k=s-21}^{40} N_k^{t-s(H)}\xi^{t-s(H)}n_k^{t-s(H)} & (s = 40,41,\cdots,61)' \end{cases} \tag{31}$$

where $RC_t^{a(H)}$, $RC_t^{b(H)}$ and $RC_t^{c(H)}$ are monetary costs for childrearing when the children will belong to the low-income class same as their parent, namely, they will become high school graduates, and $RC_t^{a(U)}$ and $RC_t^{b(U)}$ are the costs when the children will belong to the high-income class different from their parent, namely, they will become university graduates.

$$GS_t^{(U)} = \rho\left[(1-m)\sum_{s=22}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)} + RC_{s,t}^{c(U)}) + m\sum_{s=22}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)})\right], \quad (29')$$

$$\begin{cases} RC_{s,t}^{a(U)} = \sum_{k=22}^{s} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s = 22,23,\cdots,40) \\ RC_{s,t}^{b(U)} = \sum_{k=22}^{40} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s = 41,42,43) \\ RC_{s,t}^{c(U)} = \sum_{k=s-21}^{40} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s = 44,45,\cdots,61) \end{cases}, \quad (30')$$

$$\begin{cases} RC_{s,t}^{a(H)} = \sum_{k=22}^{s} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s = 22,23,\cdots,39) \\ RC_{s,t}^{b(H)} = \sum_{k=s-17}^{40} N_k^{t-s(U)}\xi^{t-s(U)}n_k^{t-s(U)} & (s = 40,41,\cdots,57) \end{cases}, \quad (31')$$

where $RC_t^{a(U)}$, $RC_t^{b(U)}$ and $RC_t^{c(U)}$ are financial costs for childrearing when the parent is 22 to 61 years old. Once the parent becomes 62 years old, the cost does not exist because all children are independent from their parent.

The total spending on the pension benefit to generations of age ST and above is represented by

$$B_t = B_t^{(H)} + B_t^{(U)}, \quad (32)$$

where $B_t^{(H)}$ and $B_t^{(U)}$ are the expenditure for the two income classes:

$$B_t^{(H)} = \sum_{s=ST}^{105} N_s^{t-s(H)} b_s^{t-s(H)}, \quad (33)$$

$$B_t^{(U)} = \sum_{s=ST}^{105} N_s^{t-s(U)} b_s^{t-s(U)}. \quad (33')$$

The total revenue from pension contributions and the total tax revenue are represented by

$$R_t = \tau^p w_t L_t, \quad (34)$$

$$T_t = \tau^w w_t L_t + \tau^r r_t AS_t + \tau_t^c AC_t + \tau^h BQ_t, \quad (35)$$

where aggregate assets supplied by households, AS_t, and aggregate consumption, AC_t, are given by

$$AS_t = AS_t^{(H)} + AS_t^{(U)}, \quad (36)$$

$$AC_t = AC_t^{(H)} + AC_t^{(U)}. \quad (37)$$

For the low-income class, aggregate assets supplied by households, $AS_t^{(H)}$, and aggregate consumption, $AC_t^{(H)}$, are given by

$$AS_t^{(H)} = \sum_{s=18}^{105} N_s^{t-s(H)} A_s^{t-s(H)}, \quad (38)$$

$$AC_t^{(H)} = \sum_{s=18}^{105} N_s^{t-s(H)} C_s^{t-s(H)} + (1-m)\sum_{s=18}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)} + RC_{s,t}^{c(H)}) + m\sum_{s=18}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)}), \quad (39)$$

where aggregate consumption consists of adult's consumption (at age 18–105 years old) and children's consumption or cost (at age zero to 17 or 21 years old).

For the high-income class, aggregate assets supplied by households, $AS_t^{(U)}$, and aggregate consumption, $AC_t^{(U)}$, are given by

$AS_t^{(U)} = \sum_{s=22}^{105} N_s^{t-s(U)} A_s^{t-s(U)}$, (38')

$AC_t^{(U)} = \sum_{s=22}^{105} N_s^{t-s(U)} C_s^{t-s(U)} + (1-m)\sum_{s=22}^{61}(RC_{s,t}^{a(U)} + RC_{s,t}^{b(U)} + RC_{s,t}^{c(U)}) +$
$m\sum_{s=22}^{57}(RC_{s,t}^{a(H)} + RC_{s,t}^{b(H)})$, (39')

where aggregate consumption consists of adult's consumption (at age 22–105 years old) and children's consumption or cost (at age zero to 21 or 17 years old).

The total sum of bequests inherited by the households and the total childrearing cost of the orphans at time t are as follows:

$$BQ_t = BQ_t^{(H)} + BQ_t^{(U)}, \quad (40)$$

$$OR_t = OR_t^{(H)} + OR_t^{(U)}. \quad (41)$$

Total population (i.e., the population aged zero to 105), the population aged 18 or 22 to 105 (i.e., independents financially), and the population aged 65 to 105 (i.e., retirees) in year t are respectively represented by

$$Z_t = Z_t^{(H)} + Z_t^{(U)}, \quad (42)$$

$$E_t = E_t^{(H)} + E_t^{(U)}, \quad (43)$$

$$O_t = O_t^{(H)} + O_t^{(U)}. \quad (44)$$

The aging rate (i.e., the old-age dependency ratio), the ratio of the population aged 65 and above to the total population, is given by O_t/Z_t. For the low-income class, the total population, the population aged 18 to 105, and the population aged 65 to 105 in year t are respectively represented by

$$Z_t^{(H)} = \sum_{k=0}^{105}\sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}, \quad (45)$$

$$E_t^{(H)} = \sum_{k=18}^{105}\sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}, \quad (46)$$

$$O_t^{(H)} = \sum_{k=65}^{105}\sum_{i=18}^{40} N_i^{t-k-i(H)} p_k^{t-k(H)} n_i^{t-k-i(H)}. \quad (47)$$

For the high-income class, the total population, the population aged 22 to 105, and the population aged 65 to 105 in year t are respectively represented by

$$Z_t^{(U)} = \sum_{k=0}^{105}\sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}, \quad (45')$$

$$E_t^{(U)} = \sum_{k=22}^{105}\sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}, \quad (46')$$

$$O_t^{(U)} = \sum_{k=65}^{105}\sum_{i=22}^{40} N_i^{t-k-i(U)} p_k^{t-k(U)} n_i^{t-k-i(U)}. \quad (47')$$

2.4. Market equilibrium

Finally, equilibrium conditions for the capital, labor and goods markets are described.

1) Equilibrium condition for the capital market

Because aggregate assets supplied by households equal the sum of real capital and net government debt,

$$AS_t = K_t + D_t. \tag{48}$$

2) Equilibrium condition for the labor market

Measured in efficiency units, because aggregate labor demand by firms equals aggregate labor supply by households,

$L_t = L_t^{(H)} + L_t^{(U)}$, (49)

where $L_t^{(H)} = \sum_{s=18}^{RE} N_s^{t-s(H)} e_s^{(H)} \{1 - l_s^{t-s(H)} - tc_s^t(n_s^{t(H)})\}$, (50)

$L_t^{(U)} = \sum_{s=22}^{RE} N_s^{t-s(U)} e_s^{(U)} \{1 - l_s^{t-s(U)} - tc_s^t(n_s^{t(U)})\}$. (50')

3) Equilibrium condition for the goods market

Because aggregate production equals the sum of private consumption, private investment and government expenditure,

$$Y_t = AC_t + \{K_{t+1} - (1 - \delta^k)K_t\} + G_t. \tag{51}$$

An iterative program is performed to obtain the equilibrium values of the above equations.

3. Simulation Analysis

3.1. Method

The simulation model presented in the previous section is solved, given the assumption that households have fundamentally perfect foresight and correctly anticipate interest, wages, the tax and contribution rates, and other factors such as the government childcare subsidies. If the tax and social security systems and other elements are determined, then the model can be solved using the Gauss–Seidel method (see Auerbach and Kotlikoff (1987) and Heer and Maußner (2005) for the computation process).

Our study assumes the transitional economy of Japan from the initial steady state in 2023 to the final steady state in 2300. For simplicity, 2023 is set as the starting year, and we simulate the demography and the economy in the following years. For the generations that were alive in 2023 and have survived in 2024, we need to pay attention to their formation of future expectations. In 2024, these generations realized that their previous expectations no longer apply and thus again maximize their remaining lifetime utility given perfect foresight. Based on the ex-post age profiles of the number of children to bear, consumption, and leisure for these generations, we calculated their lifetime utility at 18 and 22 years for the low- and high-income classes, respectively.

We assume the benchmark and alternative scenarios with the increased ratios of the government childcare subsidies to the whole childrearing cost. For alternative scenarios, the policy reform is fundamentally executed in 2024; however, in several reform scenarios, it is implemented later, such as in 2030, 2040, and 2050.

The LSRA first transfers to each household affected by the policy reform just enough resources (possibly a negative amount) to return its expected remaining lifetime utility to its pre-change level in the benchmark simulation. For each household that is alive when a policy change occurs at the end of 2023, the LSRA makes a lump sum transfer, at its age in 2024, to return its expected remaining lifetime utility to its pre-change utility level. The LSRA also makes a lump-sum transfer to each future household that enters the economy after a policy change (from 2024 onward), at its age of 18 or 22 years, to return its expected entire lifetime utility back to its pre-change level.

Note that the net present value of these transfers in 2024 across living and future households will generally not sum to 0. Thus, the LSRA makes an additional lump sum transfer to each future household so that the net present value across all transfers is 0. To illustrate, let us assume that these additional transfers are uniform across all future generations, including the low- and high-income classes. If the transfer is positive, then the change has produced extra resources after the expected remaining lifetime utility of each household has been restored to its pre-change level. In this case, we can interpret that the change has created efficiency gains, i.e., *Pareto improvements*. Conversely, if the transfer is negative, then the change has generated an efficiency loss. Thus, the total net present value of all lump sum transfers to current and future generations sums to 0 in 2024, satisfying the LSRA budget constraint (see Nishiyama and Smetters (2005) for further details).

3.2. Simulation cases

This study uses an extended lifecycle general equilibrium model with endogenous fertility. We investigate the quantitative effects of increases in government childcare subsidies on future demographics and individual welfare. We first examine the impacts of increases in the ratio of the childcare subsidies to the whole childrearing cost (ρ). In the benchmark simulation, the ratio is constant (ρ = 0.1) throughout the entire period. In 2024, seven policy reforms are performed; the ratio increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively. We next evaluate the effects of a delay in implementing policy reform, in which the increases in childcare subsidies are executed in 2030, 2040, and 2050, respectively. Finally, policy reform is not announced in advance in the abovementioned simulation cases; however, we introduce the cases with prior announcements for the delayed implementation cases (2030, 2040, and 2050).

We also investigate the impacts of the increases in childcare subsidies (ρ = 0.806), which achieves a population replacement level in the long run. Additionally, we consider a case with LSRA transfers for each scenario. We introduce LSRA into the above alternative scenarios to distinguish potential efficiency gains/losses from possibly offsetting changes in the welfare of different generations. The LSRA transfers produce a leveled and common welfare gain/loss for each future household, including the low- and high-income classes.

3.3. Specification of the parameters

We chose realistic parameter values for the Japanese economy based on the literature (Nishiyama and Smetters, 2005; Oguro et al., 2011; İmrohoroğlu et al., 2017; Kitao and Mikoshiba, 2020). Table 1 displays the parameter values assigned in the baseline simulation, and the data source used in the calibration. Parameter values were chosen such that the calculated values of the model's endogenous variables approached the actual data values. Table 2 presents the endogenous variables in the 2023 initial steady state. Because the simulation results depend on the model setting and the given parameters, we must be careful about the effects of any parameter changes.

3.3.1. Demography

The age-specific survival probability $q_{j+1|j}^{t}$ in Equation (1) is calculated from data estimated by the National Institute of Population and Social Security Research (2023). We used the average values for males and females on future life tables by age from 2023 until the last year for which official projections are available, 2070; after 2070, we used the 2070 life table data. For simplicity, the survival rate for the low-income class (i.e., high school graduates) is unity at 18 years old, and that for the high-income class (i.e., university graduates) is unity at 22.

Table 3 indicates the population ratio of individuals with different educational backgrounds in 2023, estimated from the Basic Survey on Wage Structure (Chingin Sensasu) by the Ministry of Health, Labour and Welfare (2024b). The population share of high school graduates (including junior high school graduates) and university graduates (including technical and junior college graduates) is 49.0% and 51.0%, respectively.

Figure 4 illustrates the age–population distribution in 2023 based on data from the Statistics Bureau of Japan (2024), denoting the population of high school and university graduates, respectively, for each age. We estimated each population of high school graduates and university graduates aged 0–105 in 2023, similar to Okamoto (2024a, 2024b). For the elderly, especially those of advanced age, the number of high school graduates exceeds that of the university graduates, whereas for the young and the middle-aged, it is approximately fifty–fifty. For those who are under 18 or 22 years old and undecided to become high school or university graduates, we assume that their population is the same i.e., fifty–fifty on the basis of Kikkawa (2009).

3.3.2. Preference parameter on the number of children

Regarding the preference parameter for children in the utility function of households, the parameter value is the same between the low-income class (i.e., high school graduates) and the high-income class (i.e., university graduates). In other words, the utility weight of the number of children relative to the consumption–leisure composite in Equations (2) and (2)' is the same between the two income classes ($\alpha^{(H)} = \alpha^{(U)}$ = 0.031141). This parameter setting is conducted after comprehensively considering several empirical studies, such as Kikkawa (2018) and Adsera (2017).

Initially, Kikkawa (2018) suggested that the low-income class tends to have more children than the high-income class. That study presents the scheduled number of children for young people aged 21 to 40, which is based on a large-scale questionnaire survey (SSM2015). Accordingly, on average, the scheduled number of children for young high school-graduate couples is 1.14, whereas it is 0.875 for young university-graduate couples. The data revealed that the low-income class has more children than the high-income class.

Conversely, some previous studies, such as Adsera (2017), revealed that such tendencies have weakened recently. Adsera (2017) investigated the effects of a possible increase in the employment and income gaps between highly educated and low-educated workers on their fertility. The results from that study suggested that educational attainment's negative fertility gradient has recently weakened in developed countries, and the gap in the number of children born between more-educated and less-educated women has shrunk. The results also suggested that rising inequality is one mechanism that could underlie this apparent fertility convergence. As some middle-income jobs seem to disappear, polarization in the labor market has increased. This change in the labor market could exert downward pressure on the fertility of medium- and less-educated couples and further flatten the educational gradient.

The empirical data derived by Kikkawa (2018) show that the fertility rate of the low-income class is higher than that of the high-income class, which is based on a reasonable rationale and has a certain validity; however, the findings of Adsera (2017) suggest that the fertility rate differences between the two income classes have recently decreased. Based on the above considerations, our model assumed that the parameter (α) related to the preference for the number of children in the households' utility function was set to the same value between the two income classes.

The parameter value determining the fertility was chosen so that the total fertility rate is 1.20 in the 2023 initial steady state, reflecting Japan's actual TFR. Consequently, the parameter value ($\alpha^{(H)}$, $\alpha^{(U)}$) is set to 0.031141; in the initial steady state, the TFR is 1.27 for the low-income class and 1.09 for the high-income class, indicating that the TFR is higher for the low-income class. A possible reason is that the utility obtained from the number of children is relatively higher for the low-income class than the high-income class because of the lower wage income per unit of labor.

3.3.3. Childrearing costs

Next, we describe how we assign parameter values for childrearing since our simulation model incorporates endogenous fertility. Based on empirical data, such as Kikkawa (2009), in our model, 70% of children from the high-income class will become high-income class households, and 70% of children from the low-income class will become low-income. In Japan, the high-income class spends more on educating their children than the low-income class because private education has a higher weight. This fact justifies the model setting that childrearing costs are proportional to the parent's lifetime income.

The Cabinet Office (2010) indicated the average annual childrearing costs for the first-born child to annual income for each age. Based on the survey in the Cabinet Office (2010), we assigned the parameter value of β (i.e., the ratio of childrearing costs to parental net lifetime income) such that the ratio of the annual net childrearing costs to annual labor income for the individual is, on average, close to 19.3%. Thus, β is assigned 0.046 (the ratio is 20.6 % for the low-income class and 18.3% for the high-income class).

The OECD (2024) presents public spending on family benefits in cash, services, and tax breaks for families as a percentage of GDP in 2019. For Japan, public spending ratios on family benefits in cash, services, and tax measures to GDP are 0.66%, 1.08%, and 0.20%, respectively.[2] We assigned the value of parameter ρ (government childcare subsidies divided by childrearing cost) to 0.1 in the benchmark case, as in Oguro et al. (2011). Consequently, the ratio of total government subsidies to national income was 1.21 % in the 2023 initial steady state.

Additionally, our model incorporated not only the monetary costs of childrearing but also the time costs. Increases in the number of children diminish the parent's available time, because of the time required for childrearing; more children to bear, more time required for childrearing. The parameter determining this relation, μ, is assigned under the simple assumption that one child required 1 h per day for childrearing.[3]

3.3.4. Age profile of labor efficiency

The age profiles of earning ability for the two income classes were estimated with data from the Basic Survey on Wage Structure (Chingin Sensasu) by the Ministry of Health, Labour and Welfare (2015–2024b) for the 2014–2023 period. The labor efficiency profiles are constructed from the Japanese data on employment, wages, and monthly work hours.

To estimate the age profiles of earnings ability, ${e_s}^{(H)}$ and ${e_s}^{(U)}$, respectively, the following equation is constructed:

$$Q_t = a_0 + a_1 A_t + a_2 A_t^2, \quad (52)$$

where Q is the average monthly cash earnings for high school-graduate workers and university-graduate workers, respectively, and A is the average age for each of the workers, including both males and females. Because bonuses account for a large part of earnings in Japan, Q includes bonuses. Using the above data, we use the ordinary least squares (OLS) method to perform estimation. Figure 5 presents the

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[2] In Japan, the ratio of total family benefits to GDP is only 1.95%, whereas it is, on average, 2.29% for the 38 OECD member countries. This shows that the level of governmental support for childrearing is considerably lower in Japan than that in other countries.

[3] Calibrating the value of parameter, μ, that determines the time cost in the model is difficult. In the 2023 initial steady state, an average number of children to which a parent of the low-income class gives birth during the period from 18 to 40 is 0.0277 per year. For a parent of the high-income class, it is 0.0287 per year during the period from 22 to 40. We simply assume that a parent's available time is 16 h per day and that the childrearing time cost for one child is 1 h per day.

results, illustrating age–earnings profiles by educational background. The figure shows age–earnings profiles for two representative agents: high school graduates and university graduates. For the high school graduates, they start to work earlier (18 years old), but their age profile of earnings is flatter with a lower level than the university graduates. For the university graduates, they start to work later (22 years old), but their age profile of earnings is steeper with a higher level.

3.3.5. Taxes and expenditures

Tax rates on labor income, capital income, and inheritances are assumed to be fixed at the current levels (6.5%, 40%, and 10%, respectively) during the entire period until 2300. Tax rates on consumption are endogenously determined to satisfy Equations (24) and (35). General government expenditures in Equation (27), except for transfers to the public pension sector (πB_t) and government subsidies to childrearing (GS_t), are proportional to national income (Y_t). The ratio of general expenditure to national income, g, is assigned 0.1 such that the endogenous tax rate on consumption is realistic and plausible in the 2023 initial steady state (i.e., 13.26%). The ratio is held constant at 0.1 throughout the entire period.

3.3.6. The public pension system

The public pension program is assumed to be a simple PAYG system similar to the current Japanese system. The benefit is assumed to comprise an earnings-related pension, although Japan's actual public pension system is two-tiered: a basic flat pension and an amount proportional to the average annual labor income for each household. General tax revenue finances half of the flat part, whereas contributions to the pension system fund both the remaining half and the entire proportional part. We assign the ratio (π) of the part financed by the tax transfer from the general account in Equation (26) as 0.25, taken from Oguro and Takahata (2013). The replacement ratio (θ) for public pension benefits in Equation (4) is equal to 40%, following Braun et al. (2009).

The age at which households start to receive public pension benefits (ST) is constant at 65 during the entire period. The compulsory retirement age (RE) is the starting age of public pension benefits (ST) minus 1. Thus, after households retire at the end of the year in which they reach compulsory retirement, they immediately start to receive pension benefits from the beginning of the next year.

3.3.7. Government deficits

Net government debt (D_t) is assumed to be proportional to national income to make our simulation feasible. The value of parameter d, which is the ratio of net public debt to national income as given in

Equation (25), is assigned based on data from the Ministry of Finance (2024) and the Cabinet Office (2024). After 2023, Japan's national income is expected to decrease as the population declines. Therefore, the assumption that net government debt is proportional to national income during the entire period implicitly implies that the government will successfully reduce future government deficits.

3.3.8. Intertemporal elasticity of substitution

Following İmrohoroğlu et al. (2017), the intertemporal elasticity of substitution (γ) in the individual utility function is set to 0.5. Our model also set the same value between the number of children and the consumption-leisure composite parameter, as in previous studies, such as Oguro, Takahata, and Shimasawa (2011) and Oguro and Takahata (2013).

3.3.9. Share parameter on consumption in utility

The value of the consumption share parameter, ϕ, in the utility function is assigned based on Nishiyama and Smetters (2005). Referring to Nishiyama and Smetters (2005), where $\phi = 0.47$, we set $\phi = 0.5$ in this paper. Consequently, in the 2023 initial steady state, an individual devotes, on average, 57.1% for the low-income class and 59.0% for the high-income class, of the available time endowment (of 16 h per day) to labor during their working years (ages 18–64 or 22–64 years).

3.3.10. Adjustment coefficient for discounting the future

The adjustment coefficient for discounting the future, δ, is set such that the capital–income ratio (K/Y) in the model approaches its plausible value, 2.5 which is estimated by Hansen and İmrohoroğlu (2016). Consequently, the adjustment coefficient is assigned 0.0001, which creates the capital–income ratio of 2.44.

3.3.11. Technological progress

The technological progress of private production is significant because it greatly influences economic growth. Thus, careful attention should be paid to our assumptions. Technological progress is assumed to be 0 in the simulation, reflecting Japan's experience during the past two or three decades (see Ihori et al., 2006).

4. Simulation Results

We analyze the effects of increases in government childcare subsidies on the future population and per-capita utility. Based on the simulation results, we first assess the impacts of increases in government childcare subsidies, discussing the mechanism behind the findings. We then evaluate the impacts of a delay in implementing the policy reform. Finally, we examine how prior announcements regarding policy reform affect population and utility.

4.1. Countermeasures to falling birthrates

Figures 6 and 7 illustrate the TFR and the total population, respectively, for the benchmark case ($\rho = 0.1$) and the policy reform cases where the ratio of government childcare subsidies to the whole childrearing cost (ρ) increase in 2024 from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively. In these reform cases, the childcare subsidies increase while consumption taxes cover the cost. In the baseline simulation, the TFR in Japan is 1.20 (the actual rate) in 2023 and gradually decreases to 1.05 in 2120. After that, it increases and reaches 1.10 in 2200. Conversely, the National Institute of Population and Social Security Research (2023) projects that the TFR will quickly recover after 2023 and reach 1.36 by 2070 (medium projection), as Figure 2 illustrates; however, this official projection may be overly optimistic. When the subsidy ratio dramatically increases from 0.1 to 0.8, the TFR will reach 1.99 in 2100, approaching Japan’s population replacement level of 2.07. This dramatic increase means that if the government covers 80% of the monetary childrearing cost, a constant total population can be almost achieved in the long run.

In the baseline simulation ($\rho = 0.1$), the total population is 124.4 million (the actual population) in 2023 and continues to decrease, reaching 39.6 million in 2100. Conversely, the National Institute of Population and Social Security Research (2023) projects that the total population will reach 62.8 million in 2100 (medium projection), as Figure 1 illustrates. This official projection may also be overly optimistic.[4] When the subsidy ratio dramatically increases from 0.1 to 0.8, the total population will reach 89.2 million in 2100.

Figures 6 and 7 suggest that increases in the ratio of childcare subsidies progressively enhance the total fertility rate and cumulatively augment the total population. This result quantitatively reveals that the population exponentially increases, which Malthus (1798) suggested. Figure 8 illustrates changes in

[4] Our simulation’s projected total population is much smaller than the National Institute of Population and Social Security Research (2023). One of the main reasons for this result is that our simulation does not account for immigrants at all, unlike the projection by the National Institute of Population and Social Security Research (2023). See Okamoto (2021) for the effects of introducing immigrants into our simulation model.

the national income for the reform cases in which the ratio of government childcare subsidies to national income (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively, from the benchmark case (ρ = 0.1). Increasing the government childcare subsidies amplifies the national income progressively in the long run, mainly because of the abovementioned positive effects on demography.

Table 4 reveals leveled welfare gains for each individual based on the simulation results using the LSRA method, regarding the reform cases in which the ratio of government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, and 0.6, respectively. The table shows that increases in childcare subsidies enhance welfare gains; an increase in the childcare subsidy ratio from 0.1 to 0.2 generates the individual's leveled welfare gain, equivalent to 8.73 million Japanese yen (JPY) (approximately 62,000 US dollars [USD] in 2023).[5] When the subsidy ratio increases substantially from 0.1 to 0.5, the individual's leveled welfare gain increases to 40.38 million JPY (approximately 287,000 USD in 2023), a considerable amount for each individual.

To summarize, increases in the ratio of government childcare subsidies progressively enhance the total fertility rate and augment the total population, increasing the national income progressively. Additionally, as childcare subsidies increase, the per-capita welfare gains progressively improve.

We next discuss why increases in the government childcare subsidies enhanced the welfare gains. Figures 9 and 10 illustrate the changes in interest rates and wage rates, respectively, from the benchmark case (ρ = 0.1) for the three reform cases where government childcare subsidies (ρ) increase in 2024 from 0.1 to 0.2, 0.5, and 0.8, respectively. Figure 10 shows that increases in the childcare subsidy significantly enhance the wage rates, one factor that improves individual welfare. Furthermore, increasing the subsidy augments the number of children born; these children later work and save for their lifecycle motives, which enhances capital stock, resulting in lower interest rates and higher wage rates; however, this situation gradually changes over time. Approximately 65 years later, these children retire and receive pension benefits from 65, dissaving their accumulated wealth. After that, interest rates turn higher around 2080; however, after around 2090, interest rates become lower again, compared with the benchmark case

[5] The Cabinet Office (2024) estimated that the GDP of Japan in 2023 was 559.24 trillion yen. According to data from the Ministry of Internal Affairs and Communications (2024), the number of the people aged 20–64 years was 58.80 million in 2023. We calculated the income per worker using these data and also derived the value for national GDP in 2023 in our model, yielding a conversion rate between actual amounts of yen and values in the model. Consequently, in 2023, unity in the model corresponded to 4.93095 million yen.

(ρ = 0.1). This phenomenon is caused by increases in savings for the newly born second generation, namely, the children of increased generations (parents) born by the policy reform.

Figures 11 and 12 illustrate the transition of consumption tax rates and contribution rates to the pension scheme, respectively, for the benchmark case (ρ = 0.1) and the reform cases where the government childcare subsidies (ρ) increase from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively, in 2024. Consumption taxes finance increases in childcare subsidies because our model assumes that consumption tax rates are endogenous valuables for maintaining the tax revenue. If the subsidy ratio increases dramatically from 0.1 to 0.8, the consumption tax rates will be approximately 35% in the long run; the rates will be substantially high, although this extreme case can ultimately attain almost the population replacement level. Figure 7 shows that increases in the childcare subsidy enhance the total population at an accelerated pace, increasing the ratio of the working generations to the older generations. As Figure 12 illustrates, increases in the working generations reduce the contribution rates to the pension scheme, which is one factor that improves individual welfare.

4.2. Delay in policy reform implementation

Next, we evaluate the effects of a delay in implementing countermeasures to falling birthrates in Japan. All policy reforms of the countermeasures mentioned in the previous subsection will be executed in 2024, the next year of the 2023 initial steady state. We now address the impacts of the delayed timing (2030, 2040, and 2050) of implementing the policy reforms on the future total population and the per-capita utility. The announcements of these policy reforms are conducted in the year when the policy reform is implemented (2030, 2040, and 2050) because we do not consider prior announcements of these reforms here. Figure 13 illustrates the transition of the total population for the cases of different implementation times for increases in government childcare subsidies, which produce a constant population in the long run. Increasing the ratio of government childcare subsidies (ρ) from 0.1 to 0.806 can bring about a constant population in the long run. The increased subsidy ratio of 0.806 creates a constant total population after around 2120. While a constant total population is attained, the TFR maintains around 2.01 in the model.[6]

[6] According to data from the National Institute of Population and Social Security Research (2024), the TFR of the population replacement level in Japan was 2.07 in 2022, while it is 2.01 in our simulation model. These values are relatively close, reflecting our realistic and plausible model setting. The gap between them arises mainly from our model assumption that the mortality rate, from birth to the age (18 or 22) of entry into the economy as a decision-making unit, is zero for simplicity.

When this policy reform is implemented in 2024, the total population will settle at 93.7 million in the long run. When the reform implementation is delayed to 2030 and 2040, the population reaches 84.2 million and 69.5 million, respectively, in the long run. However, if the implementation is delayed to 2050, the long-run population will be only 57.2 million; thus, delaying the implementation of the reform will substantially reduce the total population in the long run. This result occurs because Japan's total population continues to decrease, which means the number of women of childbearing age decreases. Over time, the number of these women will decrease, and the effectiveness of countermeasures to falling birthrates will be limited.

Table 4 presents leveled welfare gains for each individual for five reform cases where government childcare subsidy increases at different times. In all cases where the LSRA method, explained in subsection 3.1, is incorporated, policy reforms are announced in 2024 in the model. Even if the policy reform is implemented in 2030, 2040, and 2050, the prior announcements will all be performed in 2024 for the cases with the LSRA method. Table 4 suggests that delaying reform implementation substantially reduces leveled welfare gains. When the subsidy rate (ρ) increases from 0.1 to 0.5 in 2024, the per-capita welfare gain is 40.38 million JPY (approximately 287,000 USD in 2023). If implementation is delayed to 2030 and 2040, the per-capita welfare gain decreases to 39.09 million JPY and 31.61 million JPY, respectively. However, if the implementation is delayed to 2050, the leveled welfare gain severely diminishes to 17.62 million JPY (approximately 125,000 USD in 2023). This decline occurs because Japan's population continues to decrease, which means fewer women of childbearing age. Over time, the number of such women will decrease; thus, the effectiveness of this policy reform will be limited. Additionally, the merit of an earlier reform comes partly at the cost of more old generations that are not benefiting directly from increases in childcare allowances but suffer from higher consumption tax rates.

The above findings suggest that a delay in implementing countermeasures to falling birthrates in Japan reduces the favorable effects on the future population level and individual utility. These results also suggest that the earlier the countermeasures are implemented, the better the outcomes. Even if the implementation of the countermeasures is delayed to 2030 or 2040, the quantitative effect is still somewhat significant; however, if the implementation is delayed to 2050, the desirable effects caused by the reform would be significantly limited.

4.3. Prior announcements of the policy reform

Finally, we examine how prior announcements affect the policy reform of countermeasures of falling birthrates. Figure 14 illustrates the transition of the total population for cases with different timing of prior announcements when the policy reform of increases in government child subsidies (which produce a constant total population in the long run) is implemented in 2030. In these simulation cases, the ratio of the government childcare subsidies (ρ) increases from 0.1 to 0.806. The prior announcements are made one year to six years, respectively, before the 2030 implementation reform. Without prior announcements, the total population is 84.2 million in the long run. Table 5 presents changes in the total population in the long run for the six cases with prior announcements (from one year to six years in advance) from the case without prior announcements.

In the case with 1-year prior announcements (in 2029), the total population increases from 84.2 to 86.3 million (a 2.5% increase) in the long run. With 2-year and 3-year prior announcements (in 2028 and 2027), it increases from 84.2 to 88.0 million (a 4.5% increase) and 89.5 million (a 6.3% increase), respectively. Furthermore, in the case with 6-year prior announcements (in 2024), the total population increases from 84.2 to 92.6 million (a 10.0% increase). The earlier the advance notice is given, the larger the long-term total population; however, as the advance notice is conducted earlier, the incremental range of these favorable impacts gradually diminishes. The long-run population increases by 2.06 million from no announcements to 1-year prior announcements, 1.76 million from 1-year to 2-year prior announcements, and 1.49 million from 2-year to 3-year prior announcements. Thus, this result suggests that the advance notice of implementing countermeasures to falling birthrates has a quantitatively significant effect on future population levels. The quantitative effect of advance notice, immediately before the year of implementation (one year in advance) and up to a few years in advance, is especially significant.

Figures 15 and 16 illustrate the transition of the total population for cases with different timing of prior announcements when the policy reform of increases in government childcare subsidies (which produce a constant population in the long run) is implemented in 2040 and 2050, respectively. The ratio of the government childcare subsidies (ρ) increases from 0.1 to 0.806 in these simulation cases. The prior announcements are performed from one year to ten years in advance. Figures 15 and 16 reveal that the 3-year prior announcements enhance the long-run total population from 69.5 million to 74.2 million (a 6.8% increase) and from 57.2 million to 61.0 million (a 6.8% increase). Both figures suggest that the qualitative effect is similar to the 2030 implementation case. In other words, the earlier the advance notice is given, the larger the long-term total population; however, the earlier the notice is conducted, the more the incremental range of these favorable impacts diminishes, like in the 2030 implementation case.

Finally, we evaluate the impact of prior announcements on per-capita welfare. Table 6 presents changes in the individual lifetime utility, defined by Equations (2) and (2)’, for the 2040 policy reform cases with three different prior-announcement timings: 2030, 2035, and 2040. These reform cases increase the government childcare subsidies (ρ) from 0.1 to 0.806, producing a constant total population in the long run. Table 6 shows that the earlier the advance notice, the higher the individual lifetime utility level for both income classes. Earlier advance notice means that people are given more time to adjust, as Bütler (1999) suggested, and more information for maximizing the individual lifetime utility can help enhance the individual utility.

5. Conclusions

This paper investigated the effects of the countermeasures to falling birthrates for a model parameterized to mimic certain features of the Japanese economy. We examined this issue from two viewpoints: future demography and individual welfare. We focused on analyzing the long-term impacts on future population and per-capita welfare if the countermeasures are delayed to 2030, 2040, and 2050. Additionally, we assessed the quantitative impacts of prior announcements of policy reform. Concretely, we evaluated this issue during the transitional period, 2023–2300, using an extended lifecycle general equilibrium model with endogenous fertility. Furthermore, we introduced an LSRA to calculate the per-capita welfare and evaluate these policy reforms’ pure efficiency gains or losses.

The three main findings are as follows. First, increases in government childcare subsidies progressively enhance the total fertility rate and cumulatively augment the total population, which increases the national income progressively. Additionally, as the childcare subsidies increase, the per-capita welfare gain increases progressively.

Second, the longer the implementation of the countermeasures to falling birthrates is delayed, the lower the future total population and individual lifetime utility levels will be in the long run. Even if the implementation of the countermeasures is delayed to 2030 or 2040, the policy reform still positively affects the future population and individual utility. However, if the implementation is delayed until 2050, its desirable impacts on the future population and individual utility will be severely limited. Japan’s total population continues to decrease, which means the number of women of childbearing age is declining;

thus, the policy reform's effectiveness will diminish.

Third, prior announcements of implementing the countermeasures raise future population and individual utility levels; earlier advance notice means more time for people to adjust. The quantitative effects of advance notice are considerable, and the earlier advance notice is performed, the greater the favorable effects. In particular, the effect of advance notice immediately before the year of implementation (one year before) and up to a few years in advance is significantly large; however, the incremental range of preferable effects gradually diminishes as the advance notice is given earlier.

Finally, we mention the policy implications derived from our analysis results. The countermeasures to falling birthrates in Japan enhance the future total population and the per-capita welfare. An earlier implementation of the countermeasures would be more effective, and if it were delayed until 2050, it would be too late. Prior announcements of the implementation of countermeasures positively affect both population and welfare. The earlier the advance notice, the better the outcomes. Even if the prior announcements were conducted only one year in advance, it is imperative to keep the public well informed on the policy reform because the quantitative effect of the announcements is significant.

Appendix A: Model for the High-Income Class (University Graduates)

Here, we describe the household behavior of the high-income class household (i.e., university graduates).

A.1 Household behavior

Each agent enters the economy as a decision-making unit and starts to work at age 22 years, and lives to a maximum age of 105 years with uncertainty of death. The children aged 0–17 or 0–21 only consume, involving childrearing costs for their parent. The probability of a household born in year t, surviving until s, can be expressed by

$$p_s^{t(U)} = \prod_{j=22}^{s-1} q_{j+1|j}^t. \quad (1)'$$

Each agent who begins its economic life at age 22 chooses perfect-foresight consumption paths ($C_s^{t(U)}$), leisure paths ($l_s^{t(U)}$), and the number of born children ($n_s^{t(U)}$) to maximize a time-separable utility function of the form:

$$U^{t(U)} = \frac{1}{1-\frac{1}{\gamma}}\left[\alpha^{(U)} \sum_{s=22}^{40} p_s^{t(U)}(1+\delta)^{-(s-22)} \left(n_s^{t(U)}\right)^{1-\frac{1}{\gamma}} + (1-\alpha^{(U)}) \sum_{s=22}^{105} p_s^{t(U)}(1+\delta)^{-(s-22)} \left\{\left(C_s^{t(U)}\right)^{\varphi} \left(l_s^{t(U)}\right)^{1-\varphi}\right\}^{1-\frac{1}{\gamma}}\right]. \quad (2)'$$

where $C_s^{t(U)}$, $l_s^{t(U)}$ and $n_s^{t(U)}$ are respectively consumption, leisure and the number of children to bear (only in the first 19 periods of the life) for an agent born in year t, of age s. $\alpha^{(U)}$ is the utility weight of the number of children relative to the consumption–leisure composite.

Letting $A_s^{t(U)}$ be capital holdings for the agent born in year t, of age s, maximization of Equation (2)' is subject to a lifetime budget constraint defined by the sequence:

$$A_{s+1}^{t(U)} = \{1 + r_{t+s}(1-\tau^r)\}A_s^{t(U)} + (1-\tau^w-\tau_{t+s}^p)w_{t+s}e_s^{(U)}\{1-l_s^{t(U)}-tc_s^{t(U)}(n_s^{t(U)})\} + a_s^{t(U)} - or_s^{t(U)} + b_s^{t(U)}(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}) - (1+\tau_t^c)C_s^{t(U)} - (1-m)(1+\tau_t^c)\Phi_s^{t(U)} - m(1+\tau_t^c)\Phi_s^{t(H)}. \quad (3)'$$

There are no liquidity constraints, and thus the assets can be negative. An individual's earnings ability $e_s^{(U)}$ is an exogenous function of age, and Λ_s denotes the employment rate of age s.

The pension benefit is assumed to comprise only an earnings-related pension:

$$b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) = \begin{cases} \theta H^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) & (s \geq ST) \\ 0 & (s < ST) \end{cases}, \quad (4)'$$

where

$$H^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right) = \frac{1}{RE-21}\sum_{s=22}^{RE} w_{t+s}e_s^{(U)}\{1-l_s^{t(U)}-tc_s^t(n_s^{t(U)})\}. \quad (5)'$$

The average annual labor income for each agent is $H^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)$, and the weight coefficient of the part proportional to $H^{t(U)}$ is θ. The symbol $b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)$ in Equation (3)' signifies that the amount of public pension benefit is a function of the age profile of labor supply, $\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}$.

A parent is assumed to bear children and expend for them until they become independent of their parent, namely, during the period when they are from zero to 21 years old. Here, note that the children aged below 22 years old do not conduct an economic activity independently, and only childrearing cost for their parent arises until they become independent of their parent. The financial costs for rearing the children when the parent born in year t is s years old are represented by $\Phi_s^{t(U)}$ and $\Phi_s^{t(H)}$, which are the cost for the children who will become university graduates and high school graduates, respectively:

$$\Phi_s^{t(U)} = \begin{cases} \sum_{k=22}^{s}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s = 22,23,\cdots,40) \\ \sum_{k=22}^{40}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s = 41,42,43) \\ \sum_{k=s-21}^{40}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s = 44,45,\cdots,61) \end{cases}, \quad (6)'$$

$$\Phi_s^{t(U)} = 0 \;\; (s = 62,63,\cdots,105), \quad (7)'$$

$$\Phi_s^{t(H)} = \begin{cases} \sum_{k=22}^{s}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s = 22,23,\cdots,39) \\ \sum_{k=s-17}^{40}\xi^{t(U)}(1-\rho)n_k^{t(U)} & (s = 40,41,\cdots,57) \end{cases}, \quad (8)'$$

$$\Phi_s^{t(H)} = 0 \ \ (s = 58{,}59, \cdots, 105), \tag{9'}$$

$$\xi^{t(U)} = \beta N W^{t(U)}. \tag{10'}$$

The time cost for rearing the children when the parent born in year t is s years old is represented by

$$tc_s^t(n_s^{t(U)}) = \mu n_s^{t(U)}. \tag{11'}$$

When $BQ_t^{(U)}$ is the sum of bequests inherited by the high-income class households at time t, the bequest to be inherited by each high-income class household is defined by

$$a_s^{t(U)} = \frac{(1-\tau^h) BQ_{t+s}^{(U)}}{E_{t+s}^{(U)}}, \tag{12'}$$

where $E_t^{(U)}$ is the number of the high-income class households conducting an economic activity independently, aged 22 and above, and

$$BQ_t^{(U)} = \sum_{s=22}^{105} (N_s^{t-s-1(U))} - N_{s+1}^{t-s-1(U)}) A_{s+1}^{t-s-1(U)}. \tag{13'}$$

The number of the generation born in year t, of age s, is represented by

$$N_s^{t(U)} = p_s^{t(U)} N_0^{t(U)}. \tag{14'}$$

When $OR_t^{(U)}$ is the sum of childrearing costs incurred by the high-income class households at time t, the childrearing cost for orphans for each high-income class household is defined by

$$or_s^{t(U)} = \frac{OR_{t+s}^{(U)}}{E_{t+s}^{(U)}}, \tag{15'}$$

where

$$OR_t^{(U)} = (1-m) \sum_{s=22}^{61} (N_{s-1}^{t-s(U)} - N_s^{t-s(U)}) \Phi_s^{t-s(U)} + m \sum_{s=22}^{57} (N_{s-1}^{t-s(U)} - N_s^{t-s(U)}) \Phi_s^{t-s(H)}. \tag{16'}$$

When we consider the utility maximization problem over time for each agent, besides the flow budget constraint represented by Equation (3)', the following constraint is imposed:

$$\begin{cases} 0 \le l_s^{t(U)} \le 1 - tc_s^t(n_s^{t(U)}) (22 \le s \le RE) \\ \quad l_s^{t(U)} = 1 \ (RE + 1 \le s \le 105) \end{cases}. \tag{17'}$$

Each individual maximizes Equation (2)' subject to Equations (3)' and (17)' (see Appendix C for further details). From the utility maximization problem, the equation expressing the evolution of the number of children over time for each individual is characterized by

$$W_s^{t(U)} = \left(\frac{p_{s-1}^{t(U)}}{p_s^{t(U)}} \right) \left[\frac{1+\delta}{1 + r_{t+s}(1-\tau^r)} \right] W_{s-1}^{t(U)}, \tag{18'}$$

$$W_s^{t(U)} = \frac{\alpha^{(U)} k^{1-\frac{1}{\gamma}} (n_s^{t(U)})^{-\frac{1}{\gamma}}}{(1+\tau_{t+s}^c)\left[(1-m) \sum_{g=0}^{21} \Omega_{s,g}^t \xi^{t(U)} (1-\rho) + m \sum_{g=0}^{17} \Omega_{s,g}^t \xi^{t(U)} (1-\rho)\right]}, \tag{19'}$$

where $\Omega_{s,0}^t$=1 for $g = 0$, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g} \{1 + r_{t+s-1+k}(1-\tau^r)\} \right)^{-1}$.

Similarly, that for the consumption–leisure composite is represented by

$$V_s^{t(U)} = \left(\frac{p_{s-1}^{t(U)}}{p_s^{t(U)}}\right)\left[\frac{1+\delta}{1+r_{t+s}(1-\tau^r)}\right]V_{s-1}^{t(U)}, \qquad (20)'$$

$$V_s^{t(U)} = \frac{(1-\alpha^{(U)})\left\{(C_s^{t(U)})^{\varphi}(l_s^{t(U)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}}\varphi(C_s^{t(U)})^{\varphi-1}(l_s^{t(U)})^{1-\varphi}}{1+\tau_t^c}. \qquad (21)'$$

Appendix B: The Utility Maximization Problem for the Low-Income Class

The utility maximization problem over time for each low-income class household in Section 2 is regarded as the maximization of $U^{t(H)}$in Equation (2) subject to Equations (3) and (17). Let the Lagrange function be

$$L^{t(H)} = U^{t(H)} + \sum_{s=18}^{105}\lambda_s^{t(H)}\Big[-A_{s+1}^{t(H)} + \{1+r_{t+s}(1-\tau^r)\}A_s^{t(H)} + [1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(H)}\{1$$

$$- l_s^{t(H)} - tc_s^t(n_s^{t(H)})\} + a_s^{t(H)} - or_s^{t(H)} + b_s^{t(H)}\left(\{1-l_u^{t(H)} - tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)$$

$$- (1+\tau_{t+s}^c)C_s^{t(H)} - (1-m)(1+\tau_{t+s}^c)\Phi_s^{t(H)} - m(1+\tau_{t+s}^c)\Phi_s^{t(U)}\Big]$$

$$+\sum_{s=18}^{RE}\eta_s^{t(H)}\{1-l_s^{t(H)} - tc_s^t(n_s^{t(H)})\}$$

(B.1)

where $\lambda_s^{t(H)}$ and $\eta_s^{t(H)}$ represent the Lagrange multiplier for Equations (3) and (17), respectively.

The first-order conditions on the number of children $n_s^{t(H)}$, consumption $C_s^{t(H)}$, leisure $l_s^{t(H)}$, and assets $A_{s+1}^{t(H)}$ for s =18, 19, …, 105 can be expressed by

$$p_s^{t(H)}\alpha^{(H)}(1+\delta)^{-(s-18)}(n_s^{t(H)})^{-\frac{1}{\gamma}}$$

$$= \lambda_s^{t(H)}\Bigg\{\mu[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(H)} + (1-m)(1+\tau_{t+s}^c)\sum_{g=0}^{17}\Omega_{s,g}^t\xi^{t(H)}(1-\rho)$$

$$+m(1+\tau_{t+s}^c)\sum_{g=0}^{21}\Omega_{s,g}^t\xi^{t(H)}(1-\rho)\Big\} + \mu\sum_{k=ST}^{105}\lambda_k^{t(H)}\frac{\theta w_{t+s}e_s^{(H)}}{RE-19} + \mu\eta_s^{t(H)}, \quad (B.2)$$

where $\Omega_{s,0}^t$=1 for g =0, $\Omega_{s,g}^t = \left(\prod_{k=1}^{g}\{1+r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$,

$$p_s^{t(H)}(1-\alpha^{(H)})(1+\delta)^{-(s-18)}\left\{(C_s^{t(H)})^{\varphi}(l_s^{t(H)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}}\varphi(C_s^{t(H)})^{\varphi-1}(l_s^{t(H)})^{1-\varphi} = \lambda_s^{t(H)}(1+\tau_{t+s}^c), \quad (B.3)$$

$$p_s^{t(H)}(1-\alpha^{(H)})(1+\delta)^{-(s-18)}\left\{(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{1-\phi}\right\}^{-\frac{1}{\gamma}}(1-\phi)(C_s^{t(H)})^{\phi}(l_s^{t(H)})^{-\phi}$$

$$= \lambda_s^{t(H)} \left\{(1 - \tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(H)}\right\} + \sum_{k=ST}^{105} \lambda_k^{t(H)} \frac{\theta w_{t+s} e_s^{(H)}}{RE-19} + \eta_s^{t(H)} \quad (s \leq RE), \quad \text{(B.4)}$$

$$\lambda_s^{t(H)} = \{1 + r_{t+s}(1 - \tau^r)\} \lambda_{s+1}^{t(H)}, \qquad \text{(B.5)}$$

$$\eta_s^{t(H)} \{1 - l_s^{t(H)} - tc_s^t (n_s^{t(H)})\} = 0 \quad (s \leq RE), \qquad \text{(B.6)}$$

$$1 - l_s^{t(H)} = 0 \quad (s > RE), \qquad \text{(B.7)}$$

$$\eta_s^{t(H)} \geq 0. \qquad \text{(B.8)}$$

The combination of Equations (B.2) and (B.5) produces Equations (18) and (19). If the initial value, $n_{18}^{t(H)}$, is given, the initial value, $W_{18}^{t(H)}$, can be derived from Equation (19). If the value, $W_{18}^{t(H)}$, is specified, the value of each age, $W_s^{t(H)}$, can be derived from Equation (18), which generates the value of each age, $n_s^{t(H)}$. If the value, $n_s^{t(H)}$, is specified, the child rearing cost for lifetime is calculated, which gives the lifetime budget constraint represented by Equation (B.10).

The combination of Equations (B.3) and (B.5) produces Equations (20) and (21). If the initial value, $V_{18}^{t(H)}$, is specified, the value of each age, $V_s^{t(H)}$, can be derived from Equation (20). If $V_s^{t(H)}$ is specified, the values of consumption, $C_s^{t(H)}$, and leisure, $l_s^{t(H)}$, at each age are obtained in the method that follows.

For s =18, 19, …, *RE*, the combination of Equations (B.3) and (B.4) yields the following expression:

$$C_s^{t(H)} = \left[\frac{\varphi \left\{ (1-\tau^w - \tau_{t+s}^p) w_{t+s} e_s^{(H)} + \sum_{k=ST}^{105} \frac{\lambda_k^{t(H)}}{\lambda_s^{t(H)}} \frac{\theta w_{t+s} e_s^{(H)}}{RE-19} + \frac{\eta_s^{t(H)}}{\lambda_s^{t(H)}} \right\}}{(1-\varphi)(1+\tau_{t+s}^c)} \right] l_s^{t(H)}. \qquad \text{(B.9)}$$

If the value of $l_s^{t(H)}$ is given under $\eta_s^{t(H)}$=0, the value of $C_s^{t(H)}$ can be obtained using a numerical method, and then the value of $V_s^{t(H)}$ can be derived from Equation (21). The value of $l_s^{t(H)}$ is chosen so that the value of $V_s^{t(H)}$ obtained in the simulation is the closest to that calculated by evolution from $V_{18}^{t(H)}$ through Equation (20). If the value of $l_s^{t(H)}$ chosen is unity or higher, the value of $C_s^{t(H)}$ is obtained from Equation (21) under $l_s^{t(H)}$=1. If it is less than unity, the value of $C_s^{t(H)}$ is derived from Equation (B.9).

For s =*RE*+1, *RE*+2, …, 105, the condition of $l_s^{t(H)}$=1 leads to the following equation:

$$V_s^{t(H)} = \frac{(1-\alpha^{(H)}) \varphi (C_s^{t(H)})^{-\frac{\varphi}{\gamma} + \varphi - 1}}{1+\tau_{t+s}^c}. \qquad \text{(21)''}$$

The value of $C_s^{t(H)}$ is chosen to satisfy this equation.

From Equation (3) and the terminal condition $A_{18}^{t(H)}$=$A_{106}^{t(H)}$=0, the lifetime budget constraint for an individual (= $NW^{t(H)}$) is derived:

$$\sum_{s=18}^{RE}\Psi_s^{t(H)}[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(H)}\{1-l_s^{t(H)}-tc_s^t(n_s^{t(H)})\}+\sum_{s=ST}^{105}\Psi_s^{t(H)}b_s^{t(H)}\left(\{1-l_u^{t(H)}-tc_u^t(n_u^{t(H)})\}_{u=20}^{RE}\right)+\sum_{s=18}^{105}\Psi_s^{t(H)}(a_s^{t(H)}-or_s^{t(H)})=\sum_{s=18}^{105}\Psi_s^{t(H)}(1+\tau_{t+s}^c)C_s^{t(H)}+(1-m)\sum_{s=18}^{35}\sum_{k=18}^{s}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+(1-m)\sum_{s=36}^{40}\sum_{k=s-17}^{s}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+(1-m)\sum_{s=41}^{57}\sum_{k=s-17}^{40}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+m\sum_{s=18}^{39}\sum_{k=18}^{s}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}+m\sum_{s=40}^{61}\sum_{k=s-21}^{40}\Psi_s^{t(H)}(1+\tau_{t+s}^c)\xi^{t(H)}(1-\rho)n_k^{t(H)}, \quad \text{(B.10)}$$

where $\Psi_{18}^{t(H)}=1$ for $s=18$, $\Psi_s^{t(H)}=(\prod_{u=19}^{s}\{1+r_{t+u}(1-\tau^r)\})^{-1}$ for $s=19, 20, \ldots, 105$.

Appendix C: The Utility Maximization Problem for the High-Income Class

The utility maximization problem over time for each high-income class household in Appendix A is regarded as the maximization of $U^{t(U)}$in Equation (2)' subject to Equations (3)' and (17)'. Let the Lagrange function be

$$L^{t(U)}=U^{t(U)}+\sum_{s=22}^{105}\lambda_s^{t(U)}\Big[-A_{s+1}^{t(U)}+\{1+r_{t+s}(1-\tau^r)\}A_s^{t(U)}+[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(U)}\{1-l_s^{t(U)}-tc_s^t(n_s^{t(U)})\}+a_s^{t(U)}-or_s^{t(U)}+b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)-(1+\tau_{t+s}^c)C_s^{t(U)}-(1-m)(1+\tau_{t+s}^c)\Phi_s^{t(U)}-m(1+\tau_{t+s}^c)\Phi_s^{t(H)}\Big]+\sum_{s=22}^{RE}\eta_s^{t(U)}\left\{1-l_s^{t(U)}-tc_s^t\left(n_s^{t(U)}\right)\right\}, \quad \text{(C.1)}$$

where $\lambda_s^{t(U)}$ and $\eta_s^{t(U)}$ represent the Lagrange multiplier for Equations (3)' and (17)', respectively.

The first-order conditions on the number of children $n_s^{t(U)}$, consumption $C_s^{t(U)}$, leisure $l_s^{t(U)}$, and assets $A_{s+1}^{t(U)}$ for $s=22, 23, \ldots, 105$ can be expressed by

$$p_s^{t(U)}\alpha^{(U)}(1+\delta)^{-(s-22)}(n_s^{t(U)})^{-\frac{1}{\gamma}}=\lambda_s^{t(U)}\left\{\mu[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(U)}+(1-m)(1+\tau_{t+s}^c)\sum_{g=0}^{21}\Omega_{s,g}^t\xi^{t(U)}(1-\rho)+m(1+\tau_{t+s}^c)\sum_{g=0}^{17}\Omega_{s,g}^t\xi^{t(U)}(1-\rho)\right\}+\mu\sum_{k=ST}^{105}\lambda_k^{t(U)}\frac{\theta w_{t+s}e_s^{(U)}}{RE-21}+\mu\eta_s^{t(U)}, \quad \text{(C.2)}$$

where $\Omega_{s,0}^t=1$ for $g=0$, $\Omega_{s,g}^t=\left(\prod_{k=1}^{g}\{1+r_{t+s-1+k}(1-\tau^r)\}\right)^{-1}$,

$$p_s^{t(U)}(1-\alpha^{(U)})(1+\delta)^{-(s-22)}\left\{(C_s^{t(U)})^{\varphi}(l_s^{t(U)})^{1-\varphi}\right\}^{-\frac{1}{\gamma}}\varphi(C_s^{t(U)})^{\varphi-1}(l_s^{t(U)})^{1-\varphi}=\lambda_s^t(1+\tau_{t+s}^c), \quad \text{(C.3)}$$

$$p_s^{t(U)}(1-\alpha^{(U)})(1+\delta)^{-(s-22)}\left\{(C_s^{t(U)})^{\phi}(l_s^{t(U)})^{1-\phi}\right\}^{-\frac{1}{\gamma}}(1-\phi)(C_s^{t(U)})^{\phi}(l_s^{t(U)})^{-\phi}$$

$$= \lambda_s^{t(U)}\left\{(1-\tau^w-\tau_{t+s}^p)w_{t+s}e_s^{(U)}\right\} + \sum_{k=ST}^{105}\lambda_k^{t(U)}\frac{\theta w_{t+s}e_s^{(U)}}{RE-21} + \eta_s^{t(U)} \quad (s \le RE), \tag{C.4}$$

$$\lambda_s^{t(U)} = \{1 + r_{t+s}(1-\tau^r)\}\lambda_{s+1}^{t(U)}, \text{ (C.5)}$$

$$\eta_s^{t(U)}\{1 - l_s^{t(U)} - tc_s^t(n_s^{t(U)})\} = 0 \quad (s \le RE), \tag{C.6}$$

$$1 - l_s^{t(U)} = 0 \quad (s > RE), \tag{C.7}$$

$$\eta_s^{t(U)} \ge 0. \tag{C.8}$$

The combination of Equations (C.2) and (C.5) produces Equations (18)' and (19)'. If the initial value, $n_{22}^{t(U)}$, is given, the initial value, $W_{22}^{t(U)}$, can be derived from Equation (19)'. If the value, $W_{22}^{t(U)}$, is specified, the value of each age, $W_s^{t(U)}$, can be derived from Equation (18)', which generates the value of each age, $n_s^{t(U)}$. If the value, $n_s^{t(U)}$, is specified, the child rearing cost for lifetime is calculated, which gives the lifetime budget constraint represented by Equation (C.10).

The combination of Equations (C.3) and (C.5) produces Equations (20)' and (21)'. If the initial value, $V_{22}^{t(U)}$, is specified, the value of each age, $V_s^{t(U)}$, can be derived from equation (20)'. If $V_s^{t(U)}$ is specified, the values of consumption, $C_s^{t(U)}$, and leisure, $l_s^{t(U)}$, at each age are obtained in the method that follows.

For s =22, 23, …, *RE*, the combination of Equations (C.3) and (C.4) yields the following expression:

$$C_s^{t(U)} = \left[\frac{\varphi\left\{(1-\tau^w-\tau_{t+s}^p)w_{t+s}e_s^{(U)} + \sum_{k=ST}^{105}\frac{\lambda_k^{t(U)}}{\lambda_s^{t(U)}}\frac{\theta w_{t+s}e_s^{(U)}}{RE-21} + \frac{\eta_s^{t(U)}}{\lambda_s^{t(U)}}\right\}}{(1-\varphi)(1+\tau_{t+s}^c)}\right] l_s^{t(U)}. \tag{C.9}$$

If the value of $l_s^{t(U)}$ is given under $\eta_s^t = 0$, the value of $C_s^{t(U)}$ can be obtained using a numerical method, and then the value of $V_s^{t(U)}$ can be derived from Equation (21)'. The value of $l_s^{t(U)}$ is chosen so that the value of $V_s^{t(U)}$ obtained in the simulation is the closest to that calculated by evolution from $V_{22}^{t(U)}$ through Equation (20)'. If the value of $l_s^{t(U)}$ chosen is unity or higher, the value of $C_s^{t(U)}$ is obtained from Equation (21)' under $l_s^{t(U)}$=1. If it is less than unity, the value of $C_s^{t(U)}$ is derived from Equation (C.9).

For s =*RE*+1, *RE*+2, …, 105, the condition of $l_s^{t(U)}$=1 leads to the following equation:

$$V_s^{t(U)} = \frac{(1-\alpha^{(U)})\varphi(C_s^{t(U)})^{-\frac{\varphi}{\gamma}+\varphi-1}}{1+\tau_{t+s}^c}. \text{ (21)'''}$$

The value of $C_s^{t(U)}$ is chosen to satisfy this equation.

From Equation (3)' and the terminal condition $A_{22}^{t(U)}$=$A_{106}^{t(U)}$=0, the lifetime budget constraint for an individual (=$NW^{t(U)}$) is derived:

$$\sum_{s=22}^{RE} \Psi_s^{t(U)}[1-\tau^w-\tau_{t+s}^p]w_{t+s}e_s^{(U)}\{1-l_s^{t(U)}-tc_s^t(n_s^{t(U)})\}$$

$$+\sum_{s=ST}^{105} \Psi_s^{t(U)}b_s^{t(U)}\left(\{1-l_u^{t(U)}-tc_u^t(n_u^{t(U)})\}_{u=22}^{RE}\right)+\sum_{s=22}^{105} \Psi_s^{t(U)}(a_s^{t(U)}-or_s^{t(U)})$$

$$=\sum_{s=22}^{105} \Psi_s^{t(U)}(1+\tau_{t+s}^c)C_s^{t(U)}+(1$$

$$-m)\sum_{s=22}^{40}\sum_{k=22}^{s} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}+(1$$

$$-m)\sum_{s=41}^{43}\sum_{k=22}^{40} \Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}$$

$+(1-m)\sum_{s=44}^{61}\sum_{k=s-21}^{40}\Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}+m\sum_{s=22}^{39}\sum_{k=22}^{s}\Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}+m\sum_{s=40}^{57}\sum_{k=s-17}^{40}\Psi_s^{t(U)}(1+\tau_{t+s}^c)\xi^{t(U)}(1-\rho)n_k^{t(U)}$, (C.10)

where $\Psi_{22}^{t(U)}$=1 for s =22, $\Psi_s^{t(U)}=(\prod_{u=23}^{s}\{1+r_{t+u}(1-\tau^r)\})^{-1}$ for s =23, 24, …, 105.

References

Adsera, A. (2017) “Education and Fertility in the Context of Rising Inequality,” *Vienna Yearbook of Population Research* 15, 63–92.

Altig, D., A. J. Auerbach, L. J. Kotlikoff, Smetters, K. A., and Walliser, J. (2001) “Simulating Fundamental Tax Reform in the United States,” *American Economic Review* 91 (3), 574–595.

Aoki, R. and Vaithianthan, R. (2009) “Is Demeny Voting the Answer to Low Fertility in Japan? ” Discussion Paper Series No.435, June, Center for Intergenerational Studies, Institute of Economic Research, Hitotsubashi University (in Japanese).

Auerbach, A. J. and Kotlikoff, L. J. (1983a) “National Savings, Economic Welfare, and the Structure of Taxation” in Feldstein, M. ed., *Behavioral Simulation Methods in Tax Policy Analysis*, Chicago: University of Chicago Press, 459–493.

Auerbach, A. J. and Kotlikoff, L. J. (1983b) “An Examination of Empirical Tests of Social Security and Savings” in Helpman, E., Razin, A. and Sadka, E. eds., *Social Policy Evaluation: An Economic Perspective*, New York: Academic, 161–179.

Auerbach, A. J. and L. J. Kotlikoff (1987) *Dynamic Fiscal Policy,* Cambridge: Cambridge University Press.

Auerbach, A. J., Kotlikoff, L. J., Hagemann, R. P., and Nicoletti, G. (1989) “The Economic Dynamics of an Aging Population: The Case of Four OECD Countries,” *OECD Economic Studies*, 97–130.

Braun, R. A., Ikeda, D., and Joines, D. H. (2009) “The Saving Rate in Japan: Why It Has Fallen and Why It Will Remain Low,” *International Economic Review* 50 (1), 291–321.

Bütler, M. (1999) “Anticipation effects of looming public-pension reforms,” *Carnegie-Rochester Conference Series on Public Policy* 50, 119–159.

Hansen, G. D. and S. İmrohoroğlu (2016) “Fiscal Reform and Government Debt in Japan: A Neoclassical perspective,” *Review of Economic Dynamics* 21, 201–224.

Hayashi, F. and Prescott, E. C. (2002) “The 1990s in Japan: A Lost Decade,” *Review of Economic Dynamics* 5 (1), 206–235.

Heer, B. and Maußner, A. (2005) *Dynamic General Equilibrium Modelling*, Springer.

Homma, M., Atoda, N., Iwamoto, Y., and Ohtake, F. (1987) “Pension: Aging Society and Pension Plan” in Hamada, K., Horiuchi, A., and Kuroda, M. eds., *Macroeconomic Analysis of the Japanese Economy*, University of Tokyo Press, 149–175 (in Japanese).

Horioka, C. Y. (2021) “Is the Selfish Life-Cycle Model More Applicable in Japan, and If So, Why? A Literature Survey,” *Review of Economics of the Household* 19 (1), 157–187.

Ihori, T., Kato, R. R., Kawade, M., and Bessho, S. (2006) “Public Debt and Economic Growth in an Aging Japan,” in Kaizuka, K. and Krueger, A. O. eds., *Tackling Japan's Fiscal Challenges: Strategies to Cope with High Public Debt and Population Aging*, Palgrave Macmillan, 30–68.

Ihori, T., Kato, R. R., Kawade, M., and Bessho, S. (2011) “Health Insurance Reform and Economic Growth:

Simulation analysis in Japan," *Japan and the World Economy* 23 (4) 227–239.

İmrohoroğlu, S., S. Kitao and T. Yamada (2016) "Achieving Fiscal Balance in Japan," *International Economic Review* 57 (1), 117–154.

İmrohoroğlu, S., S. Kitao and T. Yamada (2017) "Can Guest Workers Solve Japan's Fiscal Problems?," *Economic Inquiry* 55 (3), 1287–1307.

Kato, R. (1998) "Transition to an Aging Japan: Public Pension, Savings, and Capital Taxation," *Journal of the Japanese and International Economies* 12 (3), 204–231.

Kikkawa, T. (2009) *The Society Divided by Educational Backgrounds*, Chikumashobo (in Japanese).

Kikkawa, T. (2018) *The Japanese Fragmentation: Detached Non-University Graduate Youngsters*, Kobunsha (in Japanese).

Kitao, S. (2017) "When do we Start? Pension reform in ageing Japan," *The Japanese Economic Review* 68 (1), 26–47.

Kitao, S. and M. Mikoshiba (2020) "Females, the elderly, and also males: Demographic aging and macroeconomy in Japan," *Journal of the Japanese and International Economies* 56, 101064.

Malthus, T. R. (1798) *An Essay on the Principle of Population*, London: J. Johnson.

Nakajima, T. and S. Takahashi (2017) "The optimum quantity of debt for Japan," *Journal of the Japanese and International Economies* 46, 17–26.

Nishiyama, S. and K. Smetters (2005) "Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks," *Journal of Political Economy* 113 (5), 1088–1115.

Oguro, K. and J. Takahata (2013) "Child Benefits and Macroeconomic Simulation Analyses: An Overlapping-Generations Model with Endogenous Fertility," *Public Policy Review* 9 (4), 633–659.

Oguro, K., J. Takahata, and M. Shimasawa (2011) "Child Benefit and Fiscal Burden: OLG Model with Endogenous Fertility," *Modern Economy* 2 (4), 602–613.

Okamoto, A. (2020) "Childcare Allowances and Public Pensions: Welfare and Demographic Effects in an Aging Japan," *The B.E. Journal of Economic Analysis and Policy* 20 (2), Article 20190067.

Okamoto, A. (2021) "Immigration Policy and Demographic Dynamics: Welfare Analysis of an Aging Japan," *Journal of the Japanese and International Economies* 62, Article 101168.

Okamoto, A. (2022) "Intergenerational Earnings Mobility and Demographic Dynamics: Welfare Analysis of an Aging Japan," *Economic Analysis and Policy* 74, 76–104.

Okamoto, A. (2024a) "The Optimum Quantity of Debt for an Aging Japan: Welfare and Demographic Dynamics," *The Japanese Economic Review* 75 (4) (in press).

Okamoto, A. (2024b) "Pension Reform for an Aging Japan: Welfare and Demographic Dynamics," *The SSRN Electronic Journal*. https://ssrn.com/abstract=4892318

van Groezen, B., T. Leers and L. Meijdam (2003) "Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins," *Journal of Public Economics* 87, 233–251.

Data

Cabinet Office (2010) “The Questionnaire Survey through the Internet for the Child-rearing Cost,” Cabinet Office Director-general for Policies on Cohesive Society, Japan (in Japanese).

Cabinet Office (2024) “National Accounts Statistics: GDP Statistics,” https://www.esri.cao.go.jp/jp/sna/menu.html (in Japanese).

Ministry of Finance (2024) “FY2023 budget,” https://www.mof.go.jp/english/budget/budget/index.html

Ministry of Health, Labour and Welfare (1991–2024a) “Demographic Survey.” (in Japanese) https://www.mhlw.go.jp/toukei/list/81-1a.html

Ministry of Health, Labour and Welfare (2015–2024b) “Basic Survey on Wage Structure (Chingin Sensasu),” https://www.e-stat.go.jp/ (in Japanese).

Ministry of Internal Affairs and Communications (2024) “Labour force and labour force participation rate by age group: Whole Japan,” https://www.stat.go.jp/data/roudou/longtime/03roudou.htm (in Japanese).

National Institute of Population and Social Security Research (2017) *Population Projections for Japan: 2016–2115 (Estimation in April, 2017)*, Japan (in Japanese).

National Institute of Population and Social Security Research (2023) *Population Projections for Japan: 2021–2120 (Estimation in 2023)*, Japan (in Japanese).

National Institute of Population and Social Security Research (2024) https://www.ipss.go.jp/syoushika/tohkei/Data/Popular2024/T03-09.htm (in Japanese).

OECD (2024) *OECD Family Database*, OECD, Paris. http://www.oecd.org/social/family/database.htm.

Statistics Bureau of Japan (2024) “Population Estimation,” (in Japanese) https://www.stat.go.jp/data/jinsui/index2.html#suikei

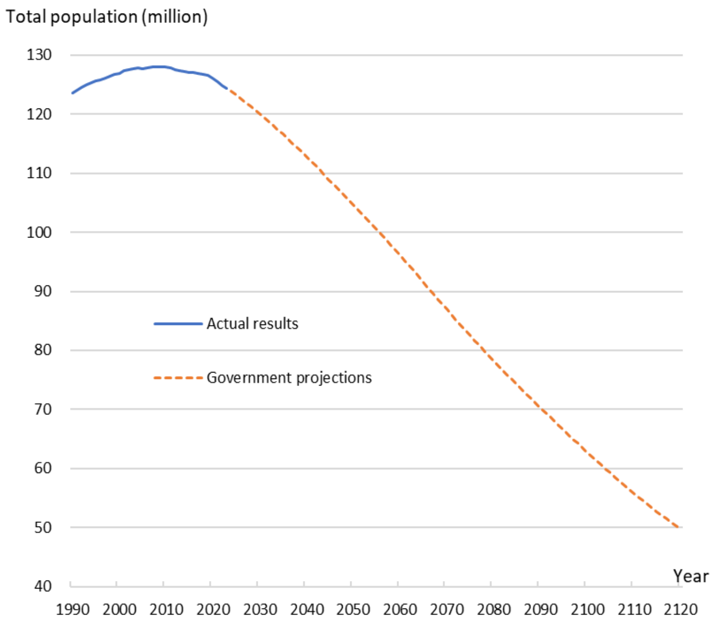

Figure 1 Total population in Japan: actual results and projections

Source: Statistics Bureau of Japan (2024) for actual results until 2023 on the total population. National Institute of Population and Social Security Research (2023) for government projections after 2023.

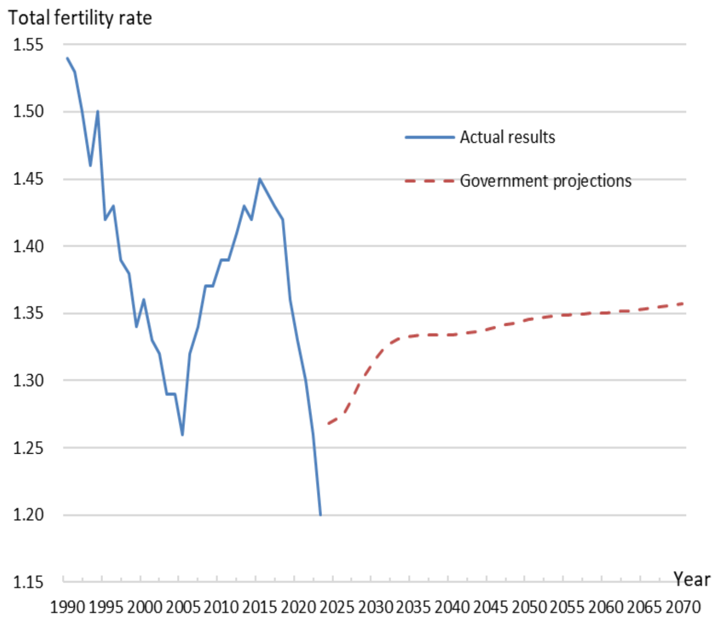

Figure 2 Total fertility rate in Japan: actual results and projections

Source: Ministry of Health, Labour and Welfare (1991–2024a) for actual results until 2023 on the total fertility rate. National Institute of Population and Social Security Research (2023) for government projections after 2023.

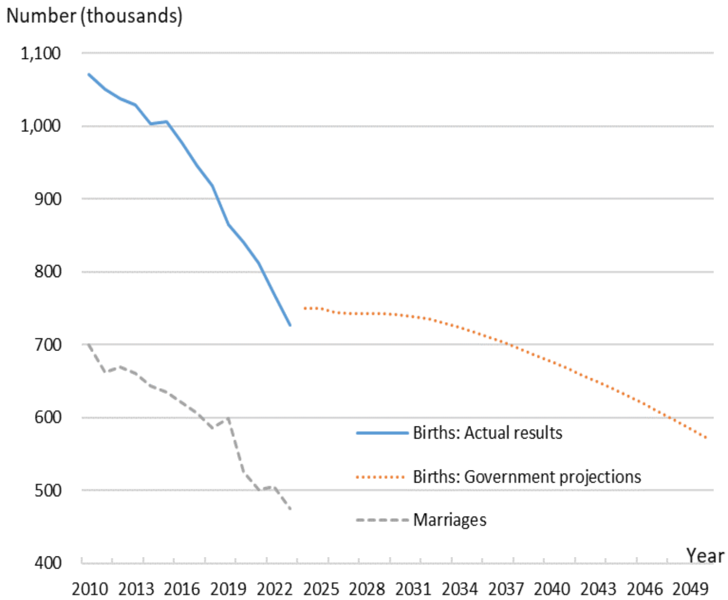

Figure 3 Number of births and marriages in Japan

Source: Ministry of Health, Labour and Welfare (2011–2024) for actual results until 2023 on births and marriages. National Institute of Population and Social Security Research (2023) for government projections on births after 2023.

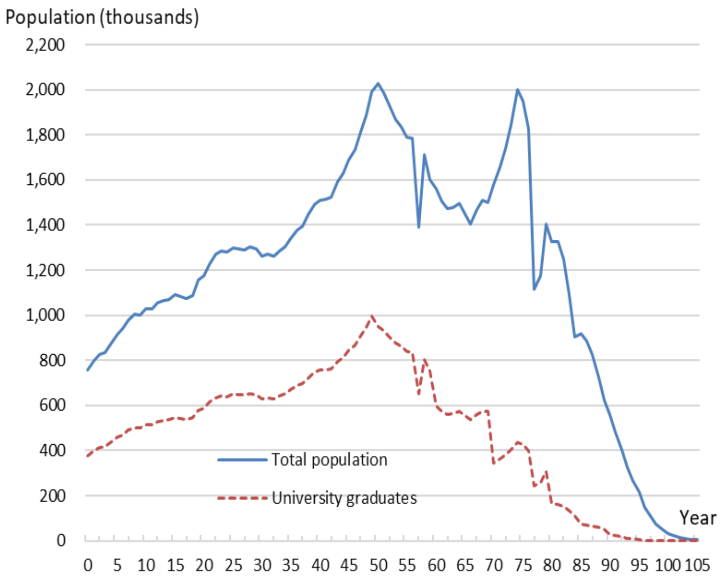

Figure 4 Age–population distribution in the 2023 initial steady state

Notes: The vertical gap between the total population and the number of university graduates is the number of high school graduates for each age. For young people unsure if they will be (just) high school graduates or university graduates, we assume 50/50.

Source: Statistics Bureau of Japan (2024)

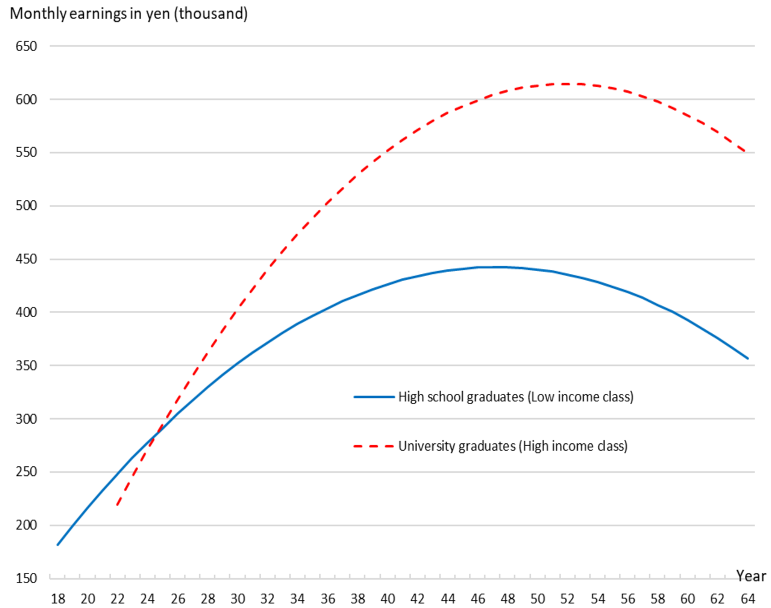

Figure 5 Age earnings profiles based on educational background

Source: The profiles are estimated from the Ministry of Health, Labour and Welfare (2015–2024b) for the 2014–2023 period.

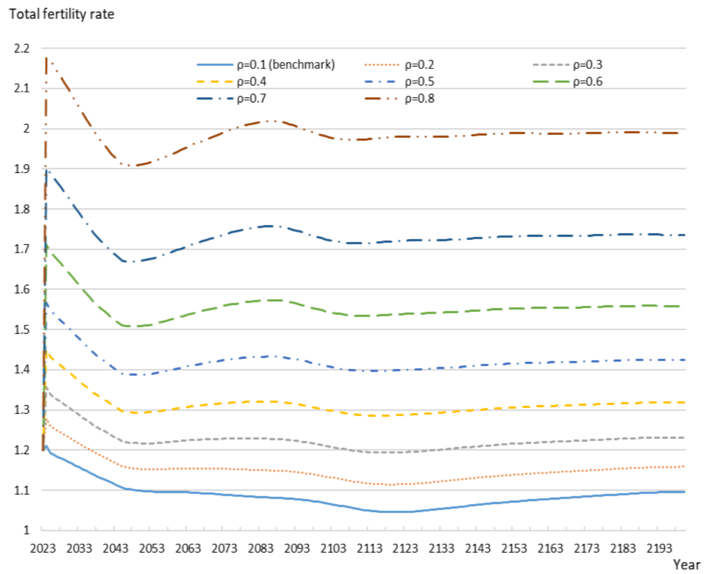

Figure 6 Transition of total fertility rates: cases of increases in government childcare subsidies

Notes: The figure shows the transition of the total fertility rates for the benchmark case (ρ = 0.1) and the 2024 policy reform cases. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively.

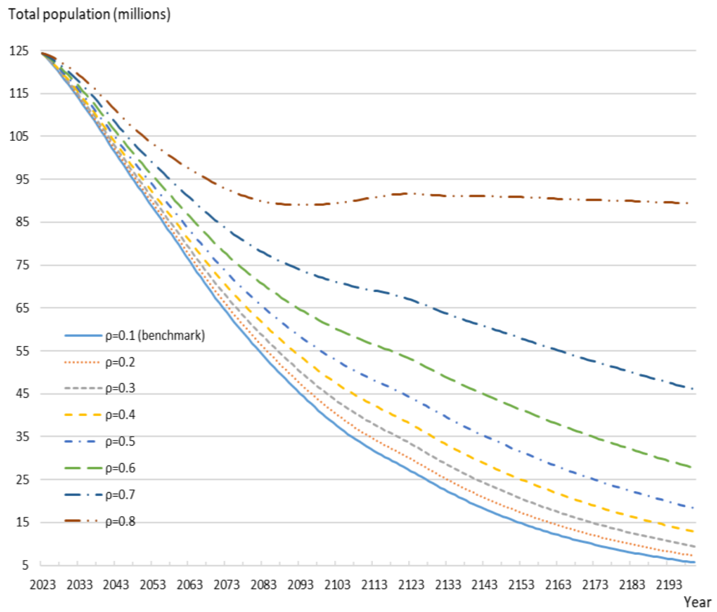

Figure 7 Transition of total population: cases of increases in government childcare subsidies

Notes: The figure shows the transition of the total fertility rates for the benchmark case ($\rho = 0.1$) and the 2024 policy reform cases. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively.

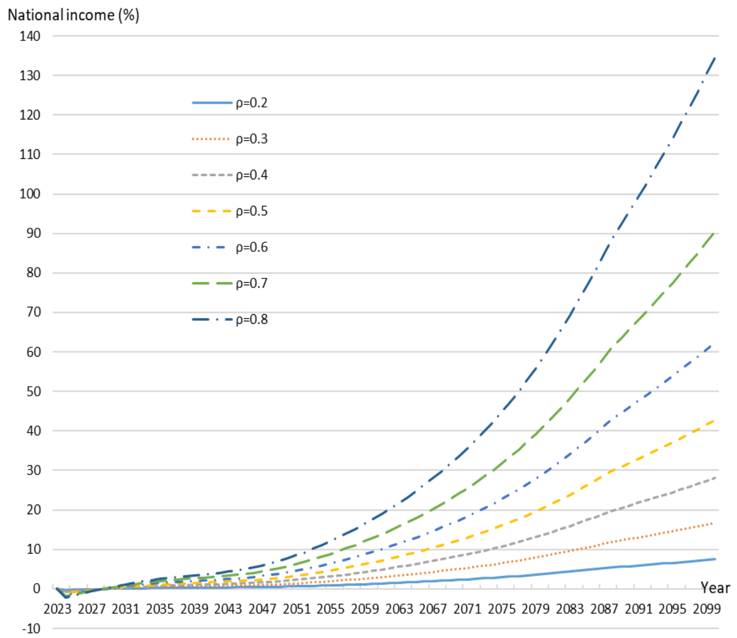

Figure 8 Transition of national income: cases of increases in government childcare subsidies

Notes: The figure shows the transition of the total fertility rates for the benchmark case ($\rho = 0.1$) and the 2024 policy reform cases. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively.

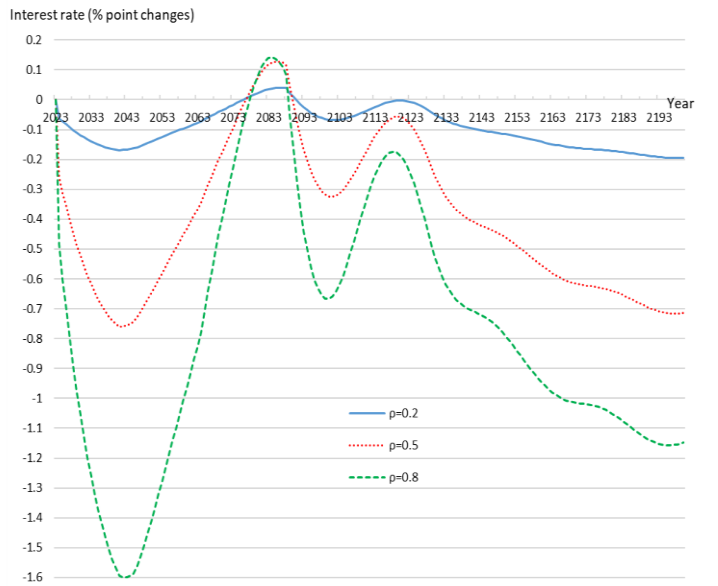

Figure 9 Transition of interest rates: cases of increases in government childcare subsidies

Notes: The figure shows changes in interest rates from the benchmark case (ρ = 0.1) and the 2024 policy reform cases.The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.5, and 0.8, respectively.

Figure 10 Transition of wage rates: cases of increases in government childcare subsidies

Notes: The figure shows changes in wage rates from the benchmark case (ρ = 0.1) and the 2024 policy reform cases. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.5, and 0.8, respectively.

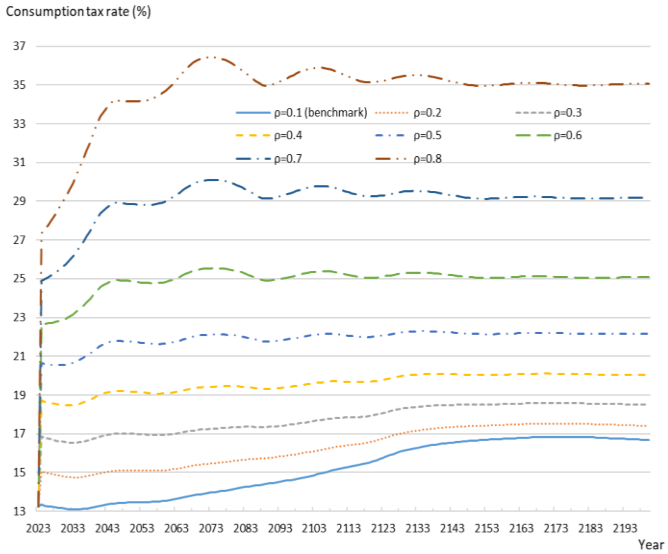

Figure 11 Transition of consumption tax rates: cases of increases in government childcare subsidies

Notes: The figure shows the transition of consumption tax rates for the benchmark case (ρ = 0.1) and the 2024 policy reform cases. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively.

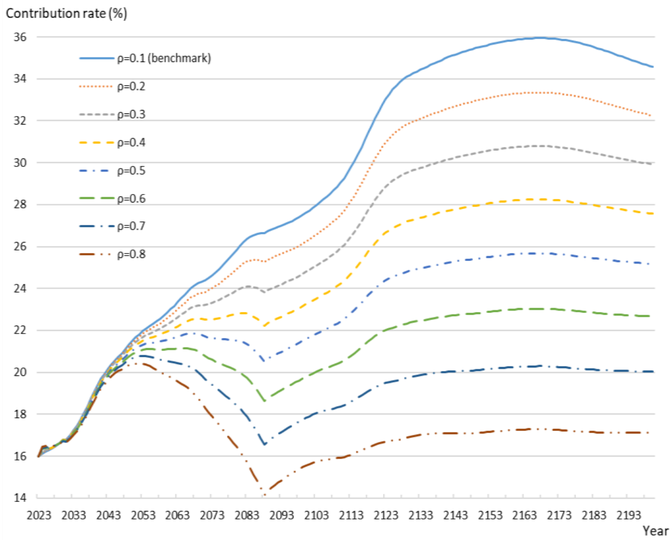

Figure 12 Transition of contribution rates: cases of increases in government childcare subsidies

Notes: The figure shows the transition of contribution rates for the benchmark case (ρ = 0.1) and the 2024 policy reform cases. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively.

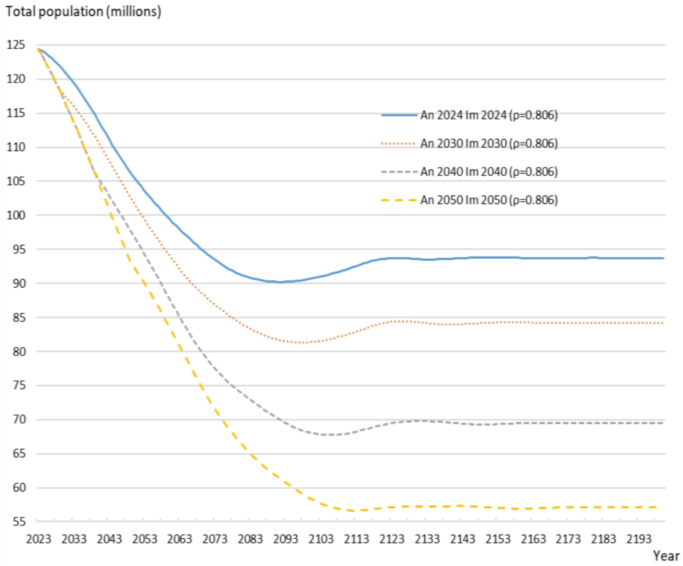

Figure 13 Transition of total population: cases of different timings of implementation for increases in government childcare subsidies, which produce a constant population in the long run

Notes: The figure shows the effect of the different timings of the policy reform implementation. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.806 on the total population in the long run.

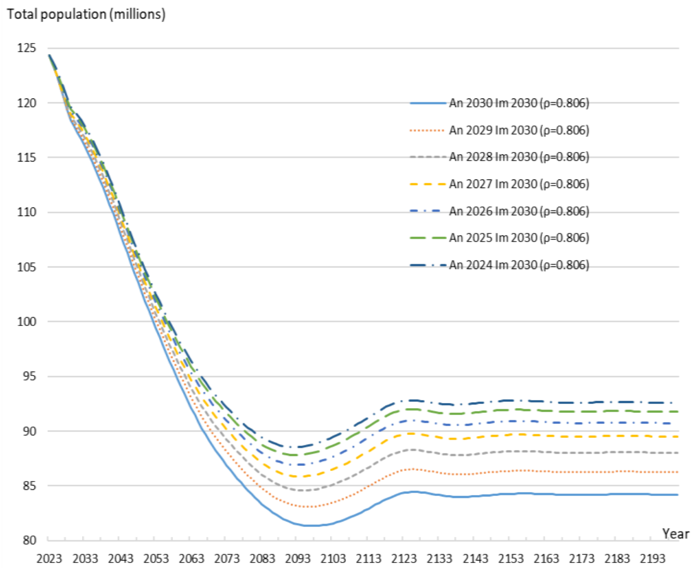

Figure 14 Transition of total population: cases of different timings of prior announcements of 2030 implementation for increases in government child subsidies, which produce a constant population in the long run

Notes: The figure shows the effect of the different timing of prior announcements for the 2030 policy reform. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.806 on the total population in the long run.

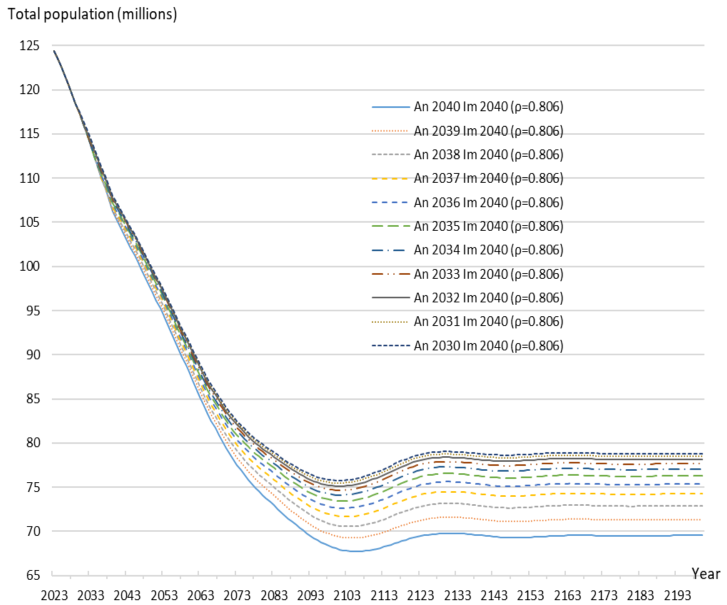

Figure 15 Transition of total population: cases of different timings of prior announcements of 2040 implementation for increases in government child subsidies, which produce a constant population in the long run

Notes: The figure shows the effect of the different timing of prior announcements for the 2040 policy reform. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.806 on the total population in the long run.

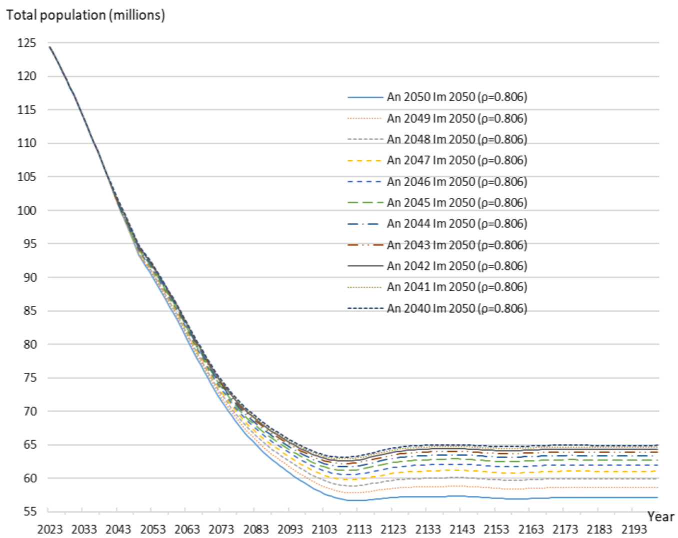

Figure 16 Transition of total population: cases of different timings of prior announcements of 2050 implementation for increases in government child subsidies which produce a constant population in the long run

Notes: The figure shows the effect of the different timing of prior announcements for the 2050 policy reform. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.806 on the total population in the long run.

Table 1 Exogenous variables for the benchmark simulation

Parameter description	Parameter value	Data source
Share parameter for consumption	φ = 0.5	Nishiyama & Smetters (2005): φ = 0.47
Weight parameter of the number of children to the consumption–leisure composite in utility	$\alpha^{(H)}=\alpha^{(U)}$= 0.031141	
Adjustment coefficient for discounting the future	δ = 0.0001	Oguro et al. (2011): δ = 0.01
Intertemporal substitution elasticity	γ = 0.5	İmrohoroğlu et al. (2017)
Ratio of government subsidies to childrearing costs	ρ = 0.1	Oguro et al. (2011): ρ = 0.1
Ratio of childrearing costs to net lifetime income	β = 0.046	
Time cost for childrearing	μ =1.7234	
Capital share in production	ε = 0.3794	İmrohoroğlu et al. (2017)
Depreciation rate	δ^k= 0.0821	İmrohoroğlu et al. (2017)
Intergenerational mobility probability between the low- and high-income classes	m = 0.3	
Tax rate on labor income	τ^w= 0.065	Kato (1998): τ^w= 0.065
Tax rate on capital income	τ^r= 0.4	Hayashi & Prescott (2002): τ^r= 0.48; İmrohoroğlu et al. (2017): τ^r= 0.35
Tax rate on inheritance	τ^h= 0.1	Kato (1998): τ^h= 0.1
Ratio of government expenditures to national income	g = 0.1	
Ratio of the part financed by tax transfer to total pension benefit	π = 0.25	Oguro & Takahata (2013): π = 0.25
Replacement ratio for public pension benefits	θ = 0.4	Braun et al. (2009): θ =0.4
Ratio of net public debt to national income	d = 1.5	İmrohoroğlu et al. (2017), Nakajima & Takahashi (2017) : d = 1.3
Compulsory retirement age	RE = 64	
Starting age for receiving public pension benefits	ST = 65	
Ratio of people aged 18 (or 22) and above to the total population	E/Z = 0.85832	
Dependency ratio (i.e., aging rate)	O/Z = 0.31550	

Table 2 Endogenous variables in the 2023 initial steady state

Parameter description	Parameter value
Interest rate, r	0.0737
Wage rate, w	1.0694
Tax rate on consumption, τ^c	0.1326
Contribution rate, τ^p	0.1599
Capital–income ratio, K/Y	2.4355
Total fertility rate (TFR)	1.2000 (low-income class 1.273; high-income class 1.092)
Ratio of net childrearing costs to annual labor income	0.2061 (low-income class) 0.1833 (high-income class)
Ratio of government childcare subsidies to national income, GS/Y	0.0121

Table 3 Population ratios among people with different educational backgrounds

<table>
<tr><th></th><th>Population (thousands)</th><th colspan="2">Population share (%)</th></tr>
<tr><td>Junior high school graduates</td><td>679.89</td><td>2.99</td><td rowspan="2">48.98</td></tr>
<tr><td>High school graduates</td><td>10,443.96</td><td>45.98</td></tr>
<tr><td>Technical and junior college</td><td>2,213.55</td><td>9.75</td><td rowspan="2">51.02</td></tr>
<tr><td>University graduates</td><td>9,374.34</td><td>41.28</td></tr>
<tr><td>Total (in year 2023)</td><td>22,711.74</td><td colspan="2">100</td></tr>
</table>

Source: The Ministry of Health, Labour and Welfare (2024b)

Table 4 Leveled welfare gains for each individual for five cases of increased government childcare subsidies implemented at different times

(million yen)

Case	2024	2030	2040	2050
ρ = 0.2	8.734	8.618	7.217	4.151
ρ = 0.3	18.287	17.962	14.621	8.365
ρ = 0.4	28.851	28.045	22.810	12.847
ρ = 0.5	40.381	39.092	31.605	17.623
ρ = 0.6	53.447	51.514	41.339	22.803

Note: The years (2024, 2030, 2040, and 2050) indicate the year in which increases in government childcare subsidies (ρ) are implemented with the 2024 (prior) announcements.

Table 5 Changes in total population in the long run from the case without prior announcements for the cases with prior announcements

(%)

Announcement	2030	2040	2050
1 year ago	2.45	2.61	2.63
2 years ago	4.54	4.85	4.89
3 years ago	6.30	6.76	6.80
4 years ago	7.76	8.34	8.42
5 years ago	8.98	9.68	9.76
6 years ago	9.97	10.77	10.88
7 years ago		11.66	11.79
8 years ago		12.36	12.53
9 years ago		12.91	13.12
10 years ago		13.31	13.55

Notes: The year (2030, 2040, and 2050) indicates the year the following policy reform is implemented. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.806, which produces a constant population in the long run. The table shows the effect of prior announcements on the total population in the long run, compared with the case without prior announcements.

Table 6　Individual lifetime utility for the 2040 policy reform for four different announcement timing

Announcement	2024	2030	2035	2040
Low-income class	–148.803	–148.822	–148.888	–149.004
High-income class	–121.583	–121.593	–121.636	–121.710

Notes: The years (2024, 2030, 2035, and 2040) indicate the announced year of the 2040 policy reform. The ratio of the government childcare subsidies to the whole childrearing cost (ρ) increases from 0.1 to 0.806, which produces a constant population in the long run. The value of the utility presented in the table is an average value of the individual lifetime utility for all generations from the 2023 initial steady state to the 2300 final steady state.